9th Linear Algebra Workshop

Portorož, Slovenia, June 2 - June 6, 2025

Book of Abstracts

UNIVERSITY OF LJUBLJANA | FMF Faculty of Mathematics and Physics

Institute of Mathematics, Physics and Mechanics

International Linear Algebra Society

UNIVERSITÀ DEL LITORALE / UNIVERZA NA PRIMORSKEM

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FACOLTÀ DI SCIENZE MATEMATICHE, NATURALI E TECNOLOGIE INFORMATICHE / FAKULTETA ZA MATEMATIKO, NARAVOSLOVJE IN INFORMACIJSKE TEHNOLOGIJE

Dear participant of the 9th Linear Algebra Workshop (LAW)

The LAW'xx meetings are back after a short "pandemic delay", an opportunity to mark the round anniversary of the founder of these gatherings Heydar Radjavi.

We would first like to thank our sponsors:

- Faculty of Maritime Studies and Transport (FPP), University of Ljubljana,
- Faculty of Mathematics and Physics, University of Ljubljana,
- Faculty of Mathematics, Natural Sciences and Information Technologies, University of Primorska,
- International Linear Algebra Society,
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for their generous support.

We kindly invite you to a **welcome reception**, which will be held on **Monday, June 2, at 19:00.** It will take place at Obala 6, 6320 Portorož, which is the building of the Faculty of Maritime Studies and Transport, University of Ljubljana, located directly by the sea shore. All accompanying persons are also welcome.

A **guided Piran walking tour** will take place on **Wednesday, June 4, at 16:30**. The trip is free for all participants. The tour will start at **Tartini square, 6330 Piran**. You can take a nice walk along the sea to reach the starting point (from FPP it is 2.2 km). But you can also take a bus (which runs very frequently every 15 minutes).

The **conference dinner** will take place on **Wednesday, June 4, at 19:00** in the restaurant Hotel Piran, Stjenkova ulica 1, 6330 Piran. There will be meat, fish and vegeterian menus available. The cost of the dinner and some drinks are included in the registration fee. The price for an accompanying person is 50 euros. The guided Piran walking trip will end at around 18:00 nearby and then you have some time to enjoy the town.

We wish you a pleasant stay in Portorož.

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9th Linear Algebra Workshop is organized by:
Institute of Mathematics, Physics, and Mechanics, Ljubljana
University of Ljubljana, Faculty of Mathematics and Physics
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Contents

1 Program of the 9^{th} Linear Algebra Workshop

Monday, June 2

	10:30 - 11:00	***Registration***
	11:00 - 11:15	***Opening***
Invited talks	11:15 - 12:05	**Marcoux** *Around commutators of quadratic operators*
	12:10 - 13:00	**Gouveia** *The nonnegative integer rank*
	13:00 - 15:00	***Lunch break***
Working groups	15:00 - 16:30	
	16:30 - 17:00	***Coffee break***
Working groups	17:00 - 18:30	
	19:00 - 21:00	***Welcome reception***

Tuesday, June 3

Invited talk	09:00 - 09:50	**de Seguins Pazzis** *A duality method for spaces of operators*	
Contributed talks	09:50 - 10:25	**Mathieu** *A solution to Jafarian's question*	
	10:25 - 11:00	**Curto** *Truncated Moment Problems: An Introductory Survey*	
	11:00 - 11:30	***Coffee break***	
Contributed talks	11:30 - 12:05	**Peperko** *Inequalities on the joint and generalized spectral radius and their essential versions*	**Jablonski** *Subnormality of the Cauchy dual operator*
	12:05 - 12:40	**Titkos** *Lattice properties of strength functions*	**Pietrzycki** *Moments and the unique extension property*
	12:40 - 13:15	**Radić** *Maps preserving the idempotency of Jordan products on $B(X)$*	**Oliveira** *Projections on matrix and operator spaces*
	13:15 - 15:00	***Lunch break***	
Working groups	15:00 - 16:30		
	16:30 - 17:00	***Coffee** break*	
Working groups	17:00 - 18:30		

Wednesday, June 4

Invited talk	09:00 - 09:50	**Šmigoc** *Arbitrarily Finely Divisible Matrices*	
Contributed talks	09:50 - 10:25	**Furuichi** *Trace inequalities and Tsallis relative entropy*	
	10:25 - 11:00	**Ilišević** *Phase-isometries between normed spaces*	
	11:00 - 11:30	***Coffee break***	
Contributed talks	11:30 - 12:05	**Kokol Bukovšek** *Compressed commuting graphs of matrix rings*	**Stochel** *Subnormality of B-operators via (conditionally) positive definite sequences*
	12:05 - 12:40	**Orel** *Some mysteries about complementary prisms*	**Chavan** *Wold-type decomposition for weighted shifts on a rootless directed tree*
	12:40 - 13:15	**Koljančić** *The strong spectral property for some families of unicyclic graphs*	**di Dio** *Positivity preservers and their generators*
	13:15 - 15:00	***Lunch break***	
	16:30 - 18:00	***Piran walking tour***	
	19:00	***Conference dinner***	

Thursday, June 5

Invited talk	09:00 - 09:50	**Breen** *Maximum spread of graphs*	
Contributed talks	09:50 - 10:25	**Fallat** *Sufficient conditions for total positivity compounds, and Dodgson condensation*	
	10:25 - 11:00	**Fialkow** *A survey of several approaches to multivariable truncated moment problems*	
	11:00 - 11:30	***Coffee break***	
Contributed talks	11:30 - 12:05	**Stopar** *Rank of elements of semiprime algebras*	**Schötz** *Localizable preordered rings*
	12:05 - 12:40	**Promyslov** *Maps preserving a fixed rank-distance on matrix spaces*	**Brüser** *Determinantal Representations of Real Positive Polynomials*
	12:40 - 13:15	**Maksaev** *Maps preserving pencil condition for singularity on real matrices*	**Nailwal** *Moment theory approach to Gaussian Quadratures with prescribed nodes*
	13:15 - 15:00	***Lunch break***	
Working groups	15:00 - 16:30		
	16:30 - 17:00	***Coffee break***	
Working groups	17:00 - 18:30		

Friday, June 6

Invited talk	09:00 - 09:50	**Tanaka** *Geometric nonlinear classification of Banach spaces arising from Birkhoff-James orthogonality*	
Contributed talks	09:50 - 10:25	**Smertnig** *An arithmetic inverse result for semigroups of invertible matrices*	
	10:25 - 11:00	**Minculete** *Some bounds for several types of entropies and divergences*	
	11:00 - 11:30	***Coffee break***	
Contributed talks	11:30 - 12:05	**Plevnik** *Linear preservers of rank one projections*	**Vitas** *The L'vov-Kaplansky Conjecture*
	12:05 - 12:40	**Đurić** *Compressed zero-divisor graphs of matrix rings over finite fields*	**Stefanović** *Isomorphisms of Birkhoff-James orthogonality on finite-dimensional C^*-algebra*

2 Invited ILAS talk

Arbitrarily Finely Divisible Matrices

Helena Šmigoc (University College Dublin)

Abstract: A stochastic matrix A is called infinitely divisible if it has a stochastic c-th root for every natural number c. If A is also nonsingular, the corresponding Markov chain is referred to as embeddable. When A is the transition matrix for the Markov chain over the time interval t_0, a stochastic c-th root of A serves as the transition matrix for the shorter time period $\frac{t_0}{c}$.

Extending this framework, we introduce *arbitrarily finely divisible stochastic matrices* (AFD_+-matrices), which have stochastic roots for infinitely many, but not necessarily all, natural numbers c. If a Markov process has transition matrices defined over infinitesimally short time intervals, then the transition matrices for this process must be AFD_+-matrices. In this talk we will present a set of general results on AFD_+-matrices and demonstrate their application to various specific cases, including a characterization of irreducible rank-two AFD_+-matrices.

This is joint work with Priyanka Joshi.

3 Invited talks

Maximum spread of graphs

Jane Breen (Ontario Tech University, Ontario, Canada)

Abstract: Given a graph G, define the spread of the graph G to be the difference between the maximum and minimum eigenvalues of its adjacency matrix. In this talk we discuss the solution to a 20-year-old conjecture of Gregory, Hershkowitz, and Kirkland regarding the spread of graphs. Our proofs use techniques from the theory of graph limits (graphons) and numerical analysis, including a computer-assisted proof of a finite-dimensional eigenvalue problem using both interval arithmetic and symbolic computations.

The nonnegative integer rank

João Gouveia (University of Coimbra, Portugal)

Abstract: Slack matrices of polyhedral cones form an important class of nonnegative matrices. They provide canonical representations of cones and play a central role in Yannakakis' seminal result on lifts of polyhedral cones. In this talk, we generalize the concept of slack matrices to affine semigroups - a discrete analog of polyhedral cones - and present a corresponding result that relates lifts of affine semigroups to nonnegative integer factorizations of their slack matrices. This generalization motivates a renewed investigation into the nonnegative integer rank of integer matrices – a specific semiring rank studied since the 1980s. Leveraging these connections, we present new theoretical and algorithmic results on this rank.

This talk is based on past and current joint work with Amy Wiebe from UBC Okanagan.

Around commutators of quadratic operators

Laurent W. Marcoux (University of Waterloo, Ontario, Canada)

Abstract: In this talk we shall discuss the norm-closures of the set of commutators of two classes of Hilbert space operators satisfying a quadratic equation, namely the class of idempotent operators and the set of square-zero operators. This is based upon joint work with Heydar Radjavi (University of Waterloo, Canada) and Yuanhang Zhang (Jilin University, China)

A duality method for spaces of operators

Clément de Seguins Pazzis (Université de Versailles-Saint-Quentin-en-Yvelines, France)

Abstract:

The topic of spaces of matrices with special properties was initiated by Dieudonné (spaces of non-singular square matrices), Gerstenhaber (spaces of nilpotent matrices) and Flanders (spaces of matrices with rank at most r) in the middle of the last century, and it has really started flourishing in the 1980's.

In this talk, I will evoke recent successes of the operator-vector duality method to connect various issues on spaces of operators with the now well-established theory of vector spaces of bounded rank matrices. In this theory, a pivotal role is played by Atkinson's theorem on primitive spaces of bounded rank matrices [1], a gem the potential of which has yet to be fully extracted.

Recent spectacular applications of this method include minimal locally linearly dependent spaces of operators [2], spaces of operators with no nonzero fixed vector [3], spaces of real diagonalisable matrices [4], real linear subspaces of complex diagonalisable matrices, spaces of triangularizable matrices over non-quadratically closed fields [5], and spaces of matrices with limited number of eigenvalues.

References

[1] M.D. Atkinson, Primitive spaces of matrices of bounded rank II. J. Austral. Math. Soc. (Ser. A) **34** (1983) 306–315.

[2] C. de Seguins Pazzis, Local linear dependence seen through duality II. Linear Algebra Appl. **462** (2014) 133–185.

[3] C. de Seguins Pazzis, From primitive spaces of bounded rank matrices to a generalized Gerstenhaber theorem. Quart. J. Math. **65-2** (2014) 319–325.

[4] C. de Seguins Pazzis, From trivial spectrum subspaces to spaces of diagonalisable real matrices. Math. Proc. R. Ir. Acad. **118-A** (2018) 5–8.

[5] C. de Seguins Pazzis, Spaces of triangularizable matrices. arXiv preprint, http://arxiv.org/abs/2410.07942

Geometric nonlinear classification of Banach spaces arising from Birkhoff-James orthogonality

Ryotaro Tanaka (Tokyo University of Science, Japan)

Abstract: As was shown by Koldobsky and Blanco-Turnšek, a linear map between normed spaces preserving Birkhoff-James orthogonality in one direction is a scalar multiple of an isometry. In particular, if there exists a linear Birkhoff-James isomorphism between normed spaces, that is, a linear bijection preserving Birkhoff-James orthogonality in both directions, then they are isometrically isomorphic. Motivated by this result, recently, the study on nonlinear versions of Birkhoff-James isomorphisms has begun. In this talk, we review a part of main results on Birkhoff-James isomorphisms and their offshoots from the viewpoint of nonlinear classification.

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4 Contributed talks

Determinantal Representations of Real Positive Polynomials

Clemens Brüser (Technische Universität Dresden, Dresden, Germany)

Abstract: This is joint work with Mario Kummer. A positive quadratic determinantal representation of a nonnegative real ternary polynomial of degree 2d is a symmetric matrix of size d with polynomial entries of degree 2 whose determinant is f, and which evaluates to a positive semidefinite matrix in every real point. The study of such representations is a relatively recent development and was initially motivated by the study of extremal rays in the cone of nonnegative biquadratic forms. Prior to our work the (non-)existence of such representations had to be established on a case-by-case basis. One of our results, however, shows that at least for smooth ternary quartics with empty real part, there always exists a positive quadratic representation. This insight relies on the fact that the combinatorial properties of quartic curves - especially their bitangent lines - are very well understood.

Wold-type decomposition for weighted shifts on a rootless directed tree

Sameer Laxman Chavan (Indian Institute of Technology Kanpur, India)

Abstract: It is known that a weighted shift on a rootless directed tree may have a non-trivial nonanalytic part. Under some mild assumptions, it is possible to identify it with a bilateral weighted shift. Moreover, reducibility of the hyper-range is closely related to the notion of the "balanced" shift. In this talk, we present a characterization of bounded left-invertible weighted shifts on a rootless directed tree, which have Wold-type decomposition. This is a joint work with Shailesh Trivedi.

Truncated Moment Problems: An Introductory Survey

Raúl Curto (The University of Iowa)

Abstract:

We present an introduction to the truncated moment problem, based on joint work with L.A. Fialkow, S.H. Lee, H.M. Möller, S. Yoo and J. Yoon.

Inverse problems naturally occur in many branches of science and mathematics. An inverse problem entails finding the values of one or more parameters using the values obtained from observed data. A typical example of an inverse problem is the inversion of the Radon transform. Here a function (for example of two variables) is deduced from its integrals along all possible lines. This problem is intimately connected with image reconstruction for X-ray computerized tomography.

Moment problems are a special class of inverse problems. While the classical theory of moments dates back to the beginning of the 20th century, the systematic study of *truncated* moment problems began only a few years ago. In this talk we will first survey the elementary theory of truncated moment problems, and then focus on those problems with cubic column relations.

Our lecture is organized around the following topics: the classical Fibonacci sequence, truncated moment problems (TMP), the basic positivity condition, the algebraic variety of a TMP, the First Existence Criterion for TMP, the Flat Extension Theorem, localizing matrices, Riesz-Haviland for TMP, the quartic TMP, the extremal TMP, the Division Algorithm in TMP and the Sextic TMP.

Positivity preservers and their generators

Philipp di Dio (University of Konstanz, Germany)

Abstract: In this talk we present the recent results about positivitiy preserver. A linear map $T : \mathbb{R}[x_1, \ldots, x_n] \to \mathbb{R}[x_1, \ldots, x_n]$ is called a K-positivity preserver if it maps all polynomials non-negative on some closed $K \subseteq \mathbb{R}^n$ to polynomials non-negative on K again. We give a full characterization and additional properties of these maps. We also define the generator of T, i.e., a map $A : \mathbb{R}[x_1, \ldots, x_n] \to \mathbb{R}[x_1, \ldots, x_n]$ such that $T = e^A$. We show the connection between generators, solutions of partial differential equations on polynomials and see how non-negative polynomials become sum of squares under certain maps A and $T = e^A$.

Compressed zero-divisor graphs of matrix rings over finite fields

Alen Đurić (Institute of Mathematics, Physics and Mechanics)

Abstract: The zero-divisor graph of a ring is a graph whose vertices are nonzero zero-divisors and there is an edge between any two nonzero vertices whose product is zero. The compressed zero-divisor graph treats all the vertices that have the same neighbourhood as a single vertex, in order to reduce the size of the graph. However, such compression of the graph is too coarse to induce a functor from a category of rings to a category of graphs in a meaningful way. This prompted us to introduce a finer compression of the zero-divisor

graph. Using our compression, the formation of the graph naturally extends to a functor. In other words, this type of compression respects not only the zero-divisor structure of a given ring but also that of its images under ring homomorphisms. Moreover, it is an optimal such compression.

In this talk, we present this type of compression with a focus on its functorial aspects, in a view towards an ongoing generalisation thereof. We investigate correspondences between algebraic properties of a ring and combinatorial properties of its compressed zero-divisor graph, e.g. the isomorphism problem. Particular attention is given to matrix rings over finite fields, for which we show, among other results, that the zero-divisor structure essentially determines the multiplicative structure and the additive structure.

References

[1] Alen Đurić, Sara Jevđenić, and Nik Stopar, *Categorial properties of compressed zero-divisor graphs of finite commutative rings*, J. Algebra Appl. 20.5 (2021), 2150069.

[2] Alen Đurić, Sara Jevđenić, and Nik Stopar, *Compressed zero-divisor graphs of matrix rings over finite fields*, Linear Multilinear Algebra 69.11 (2021), pp. 2012–2039.

Sufficient conditions for total positivity, compounds, and Dodgson condensation

Shaun Fallat (University of Regina, Canada)

Abstract: A n-by-n matrix is called totally positive (TP) if all its minors are positive and TP_k if all of its k-by-k submatrices are TP. For an arbitrary totally positive matrix or TP_k matrix, we investigate if the rth compound $(1 < r < n)$ is in turn TP or TP_k, and demonstrate a strong negative resolution in general. Applying Dodgson's algorithm for calculating the determinant of a generic matrix, we analyze whether the associated condensed matrices are possibly totally positive or TP_k. We also show that all condensed matrices associated with a TP Hankel matrix are TP.

A survey of several approaches to multivariable truncated moment problems

Lawrence Fialkow (State University of New York at New Paltz)

Abstract: Let $\beta \equiv \beta^{(2d)} = \{\beta_i\}_{i \in \mathbb{Z}_+^N, |i| \le 2d}$ denote a real N-dimensional multisequence of degree $2d$, and let K denote a closed subset of $\mathbb{R}^N$. The Truncated K-Moment Problem (TKMP) seeks necessary and sufficient conditions for the existence of a *K-representing measure* for β, i.e., a positive Borel measure μ on $\mathbb{R}^N$, with *supp* $\mu \subseteq K$, such that $\beta_i = \int x_1^{i_1} \cdots x_N^{i_N} d\mu$ $(i \equiv (i_1, \ldots, i_N) \in \mathbb{Z}_+^N, |i| \equiv i_1 + \cdots + i_N \le 2d)$. A recent comprehensive reference is the treatise [Sch] of K. Schmüdgen. We discuss several different, but equivalent, abstract solutions to TKMP: The Flat Extension Theorem [CF1] and related extension and completion methods [Z]; The Truncated Riesz-Haviland Theorem [CF2]; and The Core Variety Theorem [BF]. We illustrate these approaches with some examples.

References

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[Z] A. Zalar, The truncated Hamburger moment problems with gaps in the index set, Integ. Equ. Oper. Theory 93(2021) 36 pp.

Trace inequalities and Tsallis relative entropy

Shigeru Furuichi (Nihon University, Japan)

Abstract: Recently, F. Hansen introduced in [1] the reduced relative entropy with contraction matrtix and studied its convexity/concavity. In this talk, we firstly discuss the bounds and inequalities for the Tsallis reduced relative entropy introduced in [2]. Secondary we talk about the operator inequalities, norm inequalities and log–majorization on the spectral geometric mean [3]. As an application of the log–majorization relation, we give the upper and lower bound of the Tsallis relative entropy.

References

[1] F. Hansen, Operator means and the reduced relative quantum en tropy, Acta Sci. Math. (Szeged), 90 (2024), 565–574

[2] S. Furuichi and F. Hansen, Bounds for the reduced relative entropies, to appear in Reports on Mathematical Physics, arXiv:2312.03778.

[3] S. Furuichi and Y. Seo, Some inequalities for spectral geometric mean with applications, Linear and Multilinear Algebra, (2024), 1—20, Published online first.

Phase-isometries between normed spaces

Dijana Ilišević (University of Zagreb)

Abstract: Let X and Y be normed spaces. A mapping $f\colon X \to Y$ is called a phase-isometry if it satisfies the functional equation

$$\{\|f(x)+f(y)\|, \|f(x)-f(y)\|\} = \{\|x+y\|, \|x-y\|\}, \qquad x, y \in X.$$

Two mappings $f, g\colon X \to Y$ are said to be phase equivalent if there exists a so-called phase function σ from X to unimodular scalars such that $f(x) = \sigma(x)g(x)$ for all $x \in X$.

If X and Y are real normed spaces then any mapping that is phase equivalent to a linear isometry is a phase-isometry. This talk is about the converse.

References

[1] D. Ilišević, M. Omladič and A. Turnšek, Phase-isometries between normed spaces, Linear Algebra Appl. 612 (2021), 99-111

Subnormality of the Cauchy dual operator

Zenon Jabłoński (Jagiellonian University, Kraków)

Abstract:
For a left invertible bounded linear operator T on a complex Hilbert space $\mathcal{H}$, the Cauchy dual T' of T is given by $T' = T(T^*T)^{-1}$. An operator T is said to be a 2-isometry, if $I - 2T^*T + T^{*2}T^2 = 0$. The Cauchy dual subnormality problem asks whether the Cauchy dual operator T' of a 2-isometry T is subnormal. The talk will be devoted to discussing the negative solution to this problem, as well as positive results obtained for selected subclasses of operators.

References

[1] A. Anand, S. Chavan, Z. J. Jabłoński, J. Stochel, *A solution to the Cauchy dual subnormality problem for* 2*-isometries*, Journal of Functional Analysis **277** (2019).

[2] A. Anand, S. Chavan, Z. J. Jabłoński, J. Stochel, *The Cauchy dual subnormality problem for cyclic 2-isometries*, Advances in Operator Theory **5** (2020), 1061-1077.

Compressed commuting graphs of matrix rings

Damjana Kokol Bukovšek (University of Ljubljana)

Abstract:
In the talk we introduce compressed commuting graph of rings. It can be seen as a compression of the standard commuting graph (with the central elements added) where we identify the vertices that generate the same subring. The compression is chosen in such a way that it induces a functor from the category of rings to the category of graphs, which means that our graph takes into account not only the commutativity relation in the ring, but also the commutativity relation in all of its homomorphic images.

We show that this compression is best possible for matrix algebras over finite fields. We consider the compressed commuting graphs of finite fields, rings of 2×2 matrices over finite fields and rings of 3×3 matrices over finite prime fields.

(This is a joint work with Ivan-Vanja Boroja, Hamid Reza Dorbidi, and Nik Stopar.)

The strong spectral property for some families of unicyclic graphs

Sara Koljančić (Faculty of Natural Sciences and Mathematics, University of Banja Luka)

Abstract:
In order to find all the possible spectra of all real symmetric matrices whose off-diagonal pattern is prescribed by the adjacencies of a given graph G, the Strong Spectral Property turned out to be of crucial importance.
The set $\mathcal{G}^{\text{SSP}}$ of all simple graphs G with the property that each symmetric matrix of the pattern of G has the Strong Spectral Property is being investigated. In this talk, we will focus on families of unicyclic graphs in $\mathcal{G}^{\text{SSP}}$. We will present a complete characterization of unicyclic graphs of girth three in $\mathcal{G}^{\text{SSP}}$. Moreover, we will show that any tadpole graph of girth at most five is in $\mathcal{G}^{\text{SSP}}$ and that the same is not valid for girth six tadpole graphs.

On the cone of polynomials preserving $n \times n$ nonnegative matrices

Raphael Loewy (Department of Mathematics, Technion-Israel Institute of Technology, Israel)

Abstract: Let n be an arbitrary positive integer. Motivated by the Nonnegative Inverse Eigenvalue Problem (NIEP), Loewy and London defined the set

$$\mathcal{P}_n = \{p \in \mathbb{R}[x] : p(A) \geq 0 \text{ for all } A \geq 0, A \in \mathbb{R}^{n,n}\}.$$

This is easily seen to be a closed, pointed convex cone and its investigation is of independent interest.

In order to restrict our investigation to finite dimensional vector spaces, we restrict the degree of the polynomials. Given a positive integer m, define

$$\mathcal{P}_{n,m} = \{p \in \mathcal{P}_n : degree(p) \leq m\}.$$

Then, identifying a polynomial with the sequence of its coefficients, $\mathcal{P}_{n,m}$ can be thought as a cone in $\mathbb{R}^{m+1}$.

It is clear that any polynomial with nonnegative coefficients is in $\mathcal{P}_n$. Moreover, Clark and Paparella showed that if $p \in \mathcal{P}_n$, then its first and last n coefficients must be nonnegative. Hence, $\mathcal{P}_{n,m}$ is a simplicial cone, for any $0 < m < 2n$. Therefore, the structure of $\mathcal{P}_{n,m}$ is of interest only for $m \geq 2n$.

Let $m \geq 2n$. Then, $\mathcal{P}_{n,m}$ contains polynomials with negative coefficients. For example, there exists $a > 0$ such that $1 + x + x^2 + \cdots + x^{n-1} - ax^n + x^{n+1} + \cdots + x^{2n} \in \mathcal{P}_{n,2n}$. It follows that the structure of $\mathcal{P}_{n,m}$ is nontrivial. We consider its face structure, and in particular the one dimensional faces, that is, the extreme rays. We show that $\mathcal{P}_{n,m}$ is not polyhedral, that is, contains infinitely many extreme rays. Additional preliminary results are obtained.

Maps preserving pencil condition for singularity on real matrices

Artem Maksaev (HSE University), Valentin Promyslov (HSE University)

Abstract:

In 1949, Dieudonné [1] proved that if $T\colon M_n(\mathbb{F}) \to M_n(\mathbb{F})$ is a linear bijection preserving the set of singular matrices, then T has the standard form on $M_n(\mathbb{F})$, i. e., $T(A) = PAQ$ or $T(A) = PA^TQ$, for all $A \in M_n(\mathbb{F})$, where P, Q are non-singular. Later, a number of authors obtained generalizations and extensions of this theorem, see for example [2, 3, 4, 5].

Afterwards, in 2020, Costara [6] generalized the result of Dieudonné for maps φ_1 and φ_2 on the algebra of $n \times n$ matrices over $\mathbb{F} = \mathbb{C}$ such that at least one of φ_1, φ_2 is either continuous or surjective and the condition

$$\det(\lambda A + B) = 0 \iff \det(\lambda\varphi_1(A) + \varphi_2(B)) = 0 \quad \forall A, B \in M_n,\ \forall\lambda \in \mathbb{F} \tag{1}$$

holds. Further, the authors of [7], by using a different technique, proved the same result for an arbitrary algebraically closed field $\mathbb{F}$ and any maps φ_1, φ_2 (not necessarily continuous or surjective) satisfying the condition (1).

We consider the same problem over the field $\mathbb{R}$ of real numbers. To obtain analogous characterization it turned out to be important to consider full-spectrum matrices, i. e., $n \times n$ matrices with exactly n distinct real eigenvalues. Namely, it is sufficient to prove that any finite set of non-singular matrices can be transformed via multiplication by a matrix to a set where all matrices are full-spectrum. In this talk, we will discuss this fact, which is of independent interest, and its connection with the initial problem.

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[1] D. J. Dieudonné, Sur une généralisation du groupe orthogonal á quatre variables, Arch. Math. 1 (1949), pp. 282–287.

[2] P. Botta, Linear maps that preserve singular and nonsingular matrices, Linear Algebra Appl. 20 (1978), pp. 45–49.

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A solution to Jafarian's question

Martin Mathieu (Queen's University Belfast)

Abstract: In joint work with Francois Schulz (University of Johannesburg) we establish Jafarian's 2009 conjecture that every additive spectrum preserving mapping from a von Neumann algebra onto a semisimple Banach algebra is a Jordan isomorphism. [1]

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Some bounds for several types of entropies and divergences

Nicuşor Minculete (Transilvania University of Braşov, Romania)

Abstract: We present some characterizations with the help of inequalities for several types of entropies and divergences. Among them we identify the following: the Shannon entropy, the Renyi entropy, the Tsallis entropy, the Kullback-Leibler divergence, the Renyi divergence, the Tsallis divergence, etc. Thus, we will obtain some bounds for several types of entropies and divergences.

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Moment theory approach to Gaussian Quadratures with prescribed nodes

Rajkamal Nailwal (Institute of Mathematics, Physics and Mechanics, Ljubljana, Slovenia)

Abstract: Let μ be a positive Borel measure on the real line and let L be the linear functional on univariate polynomials of bounded degree, defined as integration with respect to μ. In [1], the characterization of all minimal quadrature rules of μ in terms of the roots of a bivariate polynomial is given and two determinantal representations of this polynomial are established. In particular, the authors solved the question of the existence of a minimal quadrature rule with one prescribed node, leaving open the extension to more prescribed nodes [1, Problem 1]. In this talk, we present a solution to this problem using moment theory as the main tool.

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Projections on matrix and operator spaces

Lina Oliveira (Center for Mathematical Analysis, Geometry and Dynamical Systems, Department of Mathematics, Instituto Superior Técnico, University of Lisbon)

Abstract: This talk explores the characterisation of Hermitian projections and of the so-called structural projections on subspaces of rectangular matrices or, more generally, on subspaces of the space $B(H, K)$ of bounded linear operators between the complex Hilbert spaces H and K. Particular emphasis will be given to the spaces of symmetric and anti-symmetric matrices and operators. We will present some explicit formulas obtained and discuss ongoing work.

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The solution to the Loewy-Radwan conjecture

Matjač Omladič (University of Ljubljana, Slovenia)

Abstract: A seminal result of Gerstenhaber gives the maximal dimension of a linear space of nilpotent matrices. It also exhibits the structure of such a space when the maximal dimension is attained. Extensions of this result in the direction of linear spaces of matrices with a bounded number of eigenvalues started 30 years ago in Ljubljana with a joint paper with Peter Šemrl. The problem was proposed in perhaps the most general way by Professor Loewy and his student Radwan 25 years ago. The positive solution to their conjecture, jointly with Klemen Šivic, ends the problem's short but exciting life in Ljubljana again. We give the dimension of a maximal vector space of n×n matrices with no more than k ¡ n eigenvalues. We also exhibit the structure of the spaces for which this dimension is attained. There is a surprising hidden connection of this solution to Heydar Radjavi that will be revealed in the talk.

Some mysteries about complementary prisms

Marko Orel (University of Primorska & IMFM)

Abstract: A preserver of a binary relation is also referred to as a *graph homomorphism*. In the talk I will consider such preservers on a *complementary prism*, which is a graph $\Gamma\bar{\Gamma}$ that is obtained from a graph Γ and its complement $\bar{\Gamma}$ by adding a perfect matching where each its edge connects two copies of the same vertex in Γ and $\bar{\Gamma}$. In general, all finite simple graphs Γ will be considered. However, vertex-transitive complementary prisms and the complementary prisms obtained from a regular self-complementary graph Γ are the most interesting. On these graphs I will present three open problems, which concern 1) graph homomorphisms, 2) the existence of a Hamiltonian cycle, and 3) the algebraic connectivity of $\Gamma\bar{\Gamma}$. The talk will be based on papers [1, 2, 3, 4, 5].

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Inequalities on the joint and generalized spectral radius and their essential versions

Aljoša Peperko (University of Ljubljana and IMFM, Ljubljana)

Abstract:

We will present several inequalities for the generalized and the joint spectral radius and their essential versions of Hadamard (Schur) weighted geometric means of bounded sets of infinite nonnegative matrices that define operators on suitable Banach sequence spaces and of bounded sets of positive kernel operators on suitable Banach function spaces.

Joint work with B. Lins and with K. Bogdanovic.

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Moments and the unique extension property

Paweł Pietrzycki (Jagiellonian University, Kraków, Poland)

Abstract:

One of the most important concepts in mathematics and physics is the notion of a normalized positive operator valued measure. This concept was introduced in the 1940s by Naimark. Recall that a map $F\colon \mathscr{A} \to \boldsymbol{B}(\mathcal{H})$ defined on a σ-algebra $\mathscr{A}$ of subsets of a set X is said to be:

- a *positive operator valued measure* (*POV measure*) if $\langle F(\cdot)h, h\rangle$ is a positive measure for every $h \in \mathcal{H}$,
- a *semispectral measure* if F is a POV measure such that $F(X) = I$,
- a *spectral measure* if F is a semispectral measure such that $F(\varDelta)$ is an orthogonal projection for every $\varDelta \in \mathscr{A}$,

One of the features of a Borel spectral measure on $\mathbb{R}$ is the multiplicativity of the corresponding Stone-von Neumann functional calculus. In particular, if E is a Borel spectral measure on $\mathbb{R}$ with compact support, then the following identities hold

$$\Big(\int_{\mathbb{R}} xE(\mathrm{d}x)\Big)^n = \int_{\mathbb{R}} x^n E(\mathrm{d}x), \quad n = 1, 2, \ldots. \tag{2}$$

Hence, all operator moments of E are determined by the first one, and according to the spectral theorem there is a one-to-one correspondence between Borel spectral measures on $\mathbb{R}$ and their first operator moments. This is no longer true for general Borel semispectral measures on $\mathbb{R}$. It turns out, however, that the single equality in (2) with $n = 2$ guarantees spectrality.

It turns out that, from a mathematical and physical point of view, it is important to investigate the relationship between semispectral and spectral measures. In the classical von Neumann description of quantum mechanics selfadjoint operators or, equivalently, Borel spectral measures on the real line represent observables. This approach is insufficient in describing many natural properties of measurements, such as measurement inaccuracy. Therefore, in standard modern quantum theory, the generalization to semispectral measures is widely used.

In 2006 Kiukas, Lahti and Ylinen asked the following general question. *When is a positive operator measure projection valued?* In this talk, we attempt to provide a partial solution to the posed question. This talk is based on joint work with Jan Stochel.

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Linear preservers of rank one projections

Lucijan Plevnik (University of Ljubljana)

Abstract:

Let $\mathcal{H}$ be a complex Hilbert space and let $\mathcal{F}_s(\mathcal{H})$ be the real vector space of all selfadjoint finite rank operators on $\mathcal{H}$. We will present the description of linear maps on $\mathcal{F}_s(\mathcal{H})$ sending rank one projections to rank one projections. Such maps are either induced by a linear or conjugate-linear isometry on $\mathcal{H}$ or constant on the set of rank one projections.

We will also discuss linear maps $\mathcal{F}_s(\mathcal{H}) \to \mathcal{F}_s(\mathcal{K})$ sending rank one projections to projections of a fixed rank. In the case $\dim \mathcal{H} = 2$, we will present the description of such maps, including a new kind of (injective) maps additionally to the previously mentioned ones. In the case $\dim \mathcal{H} = 3$, we will show by an example that such maps may be neither injective nor constant on the set of rank one projections.

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Maps preserving a fixed rank-distance on matrix spaces

Artem Maksaev (HSE University), Valentin Promyslov (HSE University)

Abstract: The problem of characterizing maps that preserve various matrix relations has a rich history, dating back to Frobenius's result [1] on linear bijective determinant-preserving maps. A significant generalization was provided by Hua [2], who studied bijective maps on matrix spaces that preserve adjacency in both directions. This result has been generalized many times, its optimal version was obtained by Šemrl [3].

We worked on another generalization of Hua's result. Specifically, we investigate bijective maps $\varphi_1, \varphi_2 \colon M_{m\times n} \to M_{m\times n}$ over a field $\mathbb{F}$ that preserve a fixed rank-distance $k \leqslant \min(m,n)$, i. e., satisfy the condition:

$$\operatorname{rk}(A-B) = k \iff \operatorname{rk}(\varphi_1(A) - \varphi_2(B)) = k, \quad \text{for all } A, B \in M_{m\times n}.$$

When $k < \min(m,n)/2$, we not only characterize such maps on matrix spaces, but proved that such maps are isometries even on more general metric spaces that we call discrete-triangular. When $k \geqslant \min(m,n)/2 \geqslant 1$, we prove that the same characterization holds for the matrices over finite fields, except for 2×2 matrices over the field of 2 elements.

It the talk we will also discuss the implications of these results for the automorphisms of the total graph of matrix algebras, particularly over the field $\mathbb{F}_2$.

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Maps preserving the idempotency of Jordan products on $B(X)$

Gordana Radić (Faculty of Electrical Engineering and Computer Science, University of Maribor)

Abstract:

This is a joint work with T. Petek and devoted to one of the so-called preserver problems. Let $B(X)$ denote the algebra of all bounded linear operators on a complex Banach space X. As it is well known, an operator $P \in B(X)$ is idempotent if $P^2 = P$. We say that a mapping $\phi : B(X) \to B(X)$ preserves idempotency of product $*$ when $\phi(A) * \phi(B)$ is idempotent whenever $A * B$ is idempotent, for every $A, B \in B(X)$. If additionally, idempotency of $\phi(A) * \phi(B)$ also implies idempotency of $A * B$, then ϕ preserves idempotency of product $*$ in both directions. Preservers of idempotency of a product can also be seen as maps preserving the zeros of a particular polynomial.

At first, unital surjective maps preserving idempotency of the standard product on $B(X)$ were described in [1]; the results were later extended in terms of omitting the unitality assumption and considering some other products, for example, the triple Jordan product. Some connections with zero product preservers can also be established.

In this talk, we will present the research considering surjective maps $\phi : B(X) \to B(X)$ (neither assumed to be linear nor additive) that preserve idempotency of Jordan product $A * B := AB + BA$ in both directions. It turns out that the Jordan product is much more challenging and we show that the related idempotency preservers are of the standard form.

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Localizable preordered rings

Matthias Schötz (Univerza v Ljubljani)

Abstract:
This talk is based on [3, 4]. Rings are understood to be associative with unit, but not commutative in general. A preordered ring $\mathcal{R}$ is endowed with a *preordering* $\mathcal{R}^+ \subseteq \mathcal{R}$ (closed under addition and multiplication, containing all squares). If $\mathcal{R}^+ \cap (-\mathcal{R}^+) = \{0\}$, then $\mathcal{R}$ is a *partially ordered ring.* Preordered rings and partially ordered rings occur most notably in real algebraic geometry, but also some lattice ordered rings are of interest, in particular f-rings. The hermitian part of commutative *-algebras of operators often turns out to be a partially ordered ring, too.

I first give an overview over the logical relations between some of the axioms of an archimedean localizable partially ordered ring $\mathcal{R}$. Here $\mathcal{R}$ is called *archimedean* if the underlying partially ordered additive group is archimedean, and $\mathcal{R}$ is *localizable* if $\mathrm{Loc}(\mathcal{R}) \subseteq \mathcal{R}$ is sufficiently large (namely cofinal), where $\mathrm{Loc}(\mathcal{R})$ is the set of all elements $s \in 1 + \mathcal{R}^+$ with the property that $sr \in \mathcal{R}^+$ or $rs \in \mathcal{R}^+$ for some $r \in \mathcal{R}$ implies $r \in \mathcal{R}^+$. Most notably, archimedean localizable partially ordered rings are always commutative. Then I present a representation theorem by rings of almost everywhere defined continuous functions. Results of this type have already been known for a long time for f-rings, e.g. [1, 2]. The notion of localizability allows to generalize these to the not lattice-ordered setting.

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An arithmetic inverse result for semigroups of invertible matrices

Daniel Smertnig (University of Ljubljana and IMFM)

Abstract: Let K be an algebraically closed field. We consider finitely generated semigroups $S \subseteq \mathrm{GL}_d(K)$ satisfying one of two arithmetic properties:

(i) S has *finitely generated spectrum*; that is, the set of all eigenvalues of elements of S is contained in a finitely generated subgroup $\Gamma \leq K^\times$;

(ii) S satisfies the (related) *Bézivin property*.

Our main result shows that property (ii) is equivalent to the representation $S \hookrightarrow \mathrm{GL}_d(K)$ factoring through a monomial representation, and that property (i) is equivalent to $S \hookrightarrow \mathrm{GL}_d(K)$ factoring through a block-triangular representation with monomial diagonal blocks.

These results are motivated by problems concerning weighted automata. Property (ii) is a relaxation of the submultiplicative spectrum property introduced by Lambrou, Longstaff, and Radjavi in 1992. The proofs use methods from Diophantine number theory (unit equations).

Joint work with Antoni Puch (University of Warsaw).

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Isomorphisms of Birkhoff-James orthogonality on finite-dimensional C^*-algebra

Srdjan Stefanović (Faculty of Mathematics, University of Belgrade)

Abstract: We will show that a bijective map strongly preserves Birkhoff-James orthogonality on a finite-dimensional complex C^*-algebra with no 1-dimensional direct summands if and only if it is a real-linear isometry multiplied by a central-valued function with some additional properties. The structure of those maps is also determined. This is a joint work with B. Kuzma ([1]).

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Subnormality of B-operators via (conditionally) positive definite sequences

Jan Stochel (Jagiellonian University)

Abstract:

The connections between harmonic analysis on groups/semigroups [3] and operator theory provide a beautiful example of the mutual influences between different branches of mathematics. One of the more classical realizations of this relationship, in the context of groups, is the dilation theory initiated by Sz.-Nagy [6]. The approach from the semigroup perspective emerged two decades later within the framework of the theory of subnormal operators, introduced into operator theory by P. Halmos [5].

Four decades later, Agler and Stankus [1, 2], inspired by Brownian motion processes, naturally introduced the class of 2-isometries, simultaneously showing that they constitute a subtle analogue of subnormal operators in the sense that every 2-isometry admits an extension to a so-called unitary Brownian operator. In other words, a 2-isometry corresponds to a subnormal operator, while a unitary Brownian operator corresponds to a normal operator.

In recent years, unitary Brownian operators have undergone a far-reaching generalization into the so-called *B-operators*, where "B" stands for "Brown-type." These operators possess a 2×2 matrix representation with operator entries, featuring a zero in the lower-left corner [4].

The aim of my talk is to investigate the subnormality of B-operators, that is, their positive definiteness, using the language of harmonic analysis. We will focus on the case where the lower-right corner is a subnormal operator and demonstrate that, in this setting, subnormality (i.e., positive definiteness) naturally relates to conditional positive definiteness, which, in turn, can be completely characterized by the positive definiteness of the discrete Laplacian. Both positive definiteness and conditional positive definiteness are understood here in the semigroup sense.

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Rank of elements of semiprime algebras

Nik Stopar (University of Ljubljana)

Abstract: Although rank is usually associated with linear algebra and matrices, it has been defined and studied in several other settings. While the origins of such investigations can be traced back as far as the 1960s and 1970s in the works of Amitsur and Puhl, perhaps the most notable contribution is by Aupetit and Mouton in 1996, who studied rank of elements of semisimple Banach algebras. They characterized the rank via the spectrum and used analytical methods to investigate it. In 1998, Brešar and Šemrl characterized the rank in algebraic terms, which allows one to extend the its definition to general rings and algebras.

In this talk, we will discuss the rank of elements of unital semiprime algebras and demonstrate how linear algebra methods can be used to investigate its properties. We will show that algebras with many elements of finite rank also have many invertible elements. In fact, our results imply that in a unital semiprime algebra every element a of finite rank is unit regular, i.e., it can be expressed as $a = eu$, where e is an idempotent of finite rank and u is an invertible element.

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Lattice properties of strength functions

Tamás Titkos (Corvinus University and Rényi Institute)

Abstract:
We investigate an important functional representation of the cone of bounded positive semidefinite operators. It is known that the representation by strength functions turns the Löwner order into the pointwise order. However, very little is known about the structure of strength functions. We are going to show that the representation behaves naturally with the infimum and supremum operations. More precisely, we show that the pointwise minimum of two strength functions f_A and f_B is a strength function if and only if the infimum of A and B exists. The cornerstone of each argument in this talk is a recent discovery of Molnár and Ramanantoanina, namely that the strength function of the parallel sum $A : B$ (which is half of the harmonic mean) equals the parallel sum of the strength functions f_A and f_B. As a byproduct of this fact, in some special cases, we describe the strength function of the so-called (generalized) short.

This is a joint work with Andriamanankasina Ramanantoanina.

The L'vov-Kaplansky Conjecture

Daniel Vitas (University of Ljubljana, Faculty of Mathematics and Physics)

Abstract: Let F be a field and let $F\langle x_1, \ldots, x_n\rangle$ denote the free algebra in (noncommuting) variables $x_1, \ldots, x_n$; we call its elements *noncommutative polynomials*. For an algebra A and a noncommutative polynomial $f \in F\langle x_1, \ldots, x_n\rangle$ we call the set

$$f(A) = \{f(a_1, \ldots, a_n) \mid a_1, \ldots, a_n \in A\} \subseteq A$$

the *image* of f in A.

The L'vov-Kaplansky conjecture states that the image of a multilinear polynomial f, i.e., a polynomial of the form $f = \sum_{\sigma \in S_n} \lambda_\sigma x_{\sigma(1)} \ldots x_{\sigma(n)}$ for some $\lambda_\sigma \in F$, in the matrix algebra $M_k(F)$ is a vector space for any $k \in \mathbb{N}$. The conjecture holds for 2×2 matrices, but even with a lot of effort by several authors, it remains only partially solved even for 3×3 matrices (see [2] for a detailed survey article regarding this).

In the presentation I will describe the L'vov-Kaplansky conjecture in detail, present some well-known results regarding the conjecture, and a new result which solves the conjecture for polynomials of degree three [3].

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5 Working groups

Local to global properties of collections of matrices

Mitja Mastnak and Heydar Radjavi

Let X be a collection of $n \times n$ matrices with structure (e.g., a semigroup, group, or a linear space). Suppose we know that every sub-collection that is in some sense "small" (e.g., size one, two, finite) satisfies some interesting property. Does this imply some significant global property for X? One of the first results of this nature is the following observation: if every element of a group of matrices is unipotent (i.e., 1 is the only eigenvalue), then the group is simultaneusly triangularizable. One known generalization is the following: suppose that $\mathrm{tr}(ABC) = \mathrm{tr}(BAC)$ for all A, B, C in a semigroup, then it also follows that the semigroup is simultaneously trianguarizable. Not all results are necessarily about triangularizability. For example: if for all A, B in an irreducible semigroup we have that the spectrum of $AB - BA$ is real, then the semigroup is simultaneously similar to a semigroup of real matrices.

The aim of this working group is to study some open problems in the area and perhaps also come up with new open problems.

Moment problems, positive polynomials and applications

Konrad Schmüdgen and Aljaž Zalar

The *moment problem (MP)* is a classical question in analysis that has been studied since the end of the 19th century (Stieltjes, 1894). A general version of the moment problem is the following: Let E be a real vector space of continuous functions on a locally compact Hausdorff space X. When is a linear functional on E an integral with respect to some positive measure on X supported by a given closed set of X? One may ask for characterizations of the existence, uniqueness and the set of representing measures.

In the past various versions of the MP appeared in the literature. In the most interesting cases E consists of multivariate polynomials on $X = \mathbb{R}^n$. Then positive polynomials play a crucial role and there is a close interaction of the moment problem and real algebraic geometry. When the degree of the polynomials is bounded, we have the *truncated MP*. When the number of variables is infinite, the problem is usually called an *infinite dimensional MP*. When E is some unital commutative real algebra A and X the set of all characters of A, we obtain an *abstract formulation of the MP*. When E is replaced by a vector space of matrix/operator polynomials or when the functional is replaced by the operator which maps into matrices, we have *non-commutative versions of the MP*.

Nowadays, MPs find their applications in many fields such as real algebraic geometry, polynomial optimization, operator theory, probability and statistics, the theory of differential equations, statistical physics and others.

This working group will consist of talks, followed by discussions on open problems in the area, presented by participants.

Preserver problems

Chi-Kwong Li

Preserver problems concern the study of maps Φ on matrices or operators with some special properties such as $\Phi(S) \subseteq S$ for a given set S, $f(\Phi(A)) = f(A)$ for given function f, or $\Phi(A) \sim \Phi(B)$ whenever $A \sim B$ for a given relation $\sim$. The study has attracted many researchers because the subject has connections with many pure and applied areas. In particular, many preservers results have implications to other subjects, and many techniques from other subjects could be used to study preserver problems.

In this working group, we would like to bring (experienced or new) researchers interested in the topic to share results, experience, problems, and explore further connections of the topic to other areas.
