Inequalities on the spectral radius, operator norm and numerical radius of Hadamard weighted geometric mean of positive kernel operators

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In the talk several inequalities, on the spectral radius ρ, operator norm $\|\cdot\|$ and numerical radius of Hadamard products and ordinary products of non-negative matrices that define operators on sequence spaces, or of Hadamard geometric mean and ordinary products of positive kernel operators on Banach function spaces, will be presented. These inequalities generalize or refine ealier results of several authors. In particular, we show that for a Hadamard geometric mean $A^{\left(\frac{1}{2}\right)} \circ B^{\left(\frac{1}{2}\right)}$ of positive kernel operators A and B on a Banach function space L, we have

$$\rho\left(A^{\left(\frac{1}{2}\right)} \circ B^{\left(\frac{1}{2}\right)}\right) \leq \rho\left((AB)^{\left(\frac{1}{2}\right)} \circ (BA)^{\left(\frac{1}{2}\right)}\right)^{\frac{1}{2}} \leq \rho(AB)^{\frac{1}{2}}.$$

In the special case $L = L^2(X, \mu)$ we also prove that

$$\|A^{\left(\frac{1}{2}\right)} \circ B^{\left(\frac{1}{2}\right)}\| \leq \rho\left((A^*B)^{\left(\frac{1}{2}\right)} \circ (B^*A)^{\left(\frac{1}{2}\right)}\right)^{\frac{1}{2}} \leq \rho(A^*B)^{\frac{1}{2}}.$$

If time allows, we will also present some related inequalities for the generalized and joint spectral radius for bounded sets of positive kernel operators. The talk is mostly based on the preprints [1, 2].

REFERENCES

[1] A. Peperko, *Inequalities on the spectral radius, operator norm and numerical radius of Hadamard weighted geometric mean of positive kernel operators*, 2016, https://arxiv.org/abs/1612.01767

[2] A. Peperko, *Bounds on the joint and generalized spectral radius of Hadamard geometric mean of bounded sets of positive kernel operatorsbqtt n*, 2016, https://arxiv.org/abs/1612.01765