Some open questions about Kronecker quotients

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This talk will review results on the existence of different types of Kronecker quotients, and consider some open questions regarding uniform Kronecker quotients.

Consider the vector space $\mathcal{M}_{m,n}$ of $m \times n$ matrices over some field $\mathbb{F}$ and the Kronecker product $A \otimes B \in \mathcal{M}_{ms,nt}$ of matrices $A \in \mathcal{M}_{m,n}$ and $B \in \mathcal{M}_{s,t}$.

We may define a quotient operation $\oslash : \mathcal{M}_{ms,nt} \times \mathcal{M}_{s,t} \to \mathcal{M}_{m,n}$. A Kronecker quotient $\oslash$ obeys $(A \otimes B) \oslash B = A$ for all matrices A and $B \neq 0$. In particular, a *uniform Kronecker quotient* $\oslash$ is linear in its left argument and obeys

$$(A \otimes B) \oslash C = (B \oslash C)A$$

when B and C have the same size (so that $B \oslash C \in \mathbb{F}$).

For each uniform Kronecker quotient, there exists $Q : \mathcal{M}_{s,t} \to \mathcal{M}_{s,t}$ such that, for $M \in \mathcal{M}_{ms,nt}$,

$$M \oslash B = \operatorname{tr}_2((I_m \otimes Q(B))^T M)$$

where I_m is the $m \times m$ identity matrix and the partial trace tr_2 is taken over the $t \times t$ matrices in $\mathcal{M}_{m,n} \otimes \mathcal{M}_{t,t}$. This is known as a partial Frobenius product.