Krauter conjecture on permanents is true

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Permanent function is very useful in algebra and combinatorics. The central role in these investigations is played by $(0,1)$ or $(-1,1)$ matrices, which are important for the combinatorial applications of permanent. The following problem related to the permanent of $(-1,1)$ matrices was posed in 1974.

Problem 1. (Wang, [2, Problem 2].) *Let $A \in M_n(\pm 1)$ be a nonsingular matrix. Is there a decent upper bound for $|\mathrm{per}\,(A)|$?*

Kräuter in the paper [1] formulated the following conjecture which provides a possible upper bound for the values of the permanent function for matrices from $M_n(\pm 1)$ via the rank function.

Conjecture 1. (Kräuter, [1, Conjecture 5.2], 1985.) *Let $n \geq 5$, $A \in M_n(\pm 1)$ and $\mathrm{rk}\,(A) = r+1$ for some r, $0 \leq r \leq n-1$. Then $|\mathrm{per}\,(A)| \leq \mathrm{per}\,(D(n,r))$, where $D(n,r) = (d_{ij}) \in M_n(\pm 1)$ is defined by $d_{ij} = -1$ if $i = j$ and $j \in \{1,\ldots,r\}$, $d_{ij} = 1$ otherwise. The equality holds iff the matrix A can be obtained from $D(n,r)$ by the transposition, row or column permutations and multiplications of rows or columns by -1.*

We show that this conjecture is true.

This is a joint work with MIKHAIL BUDREVICH (Lomonosov Moscow State University).

REFERENCES

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[2] E.T.H. Wang, *On permanents of (+1, -1)-matrices*, Israel J. Math., **18**, 1974, 353–361.