Koopman Semigroups

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Already in the 1930s, B. O. Koopman, G. D. Birkhoff, and J. von Neumann observed that a system of nonlinear ordinary differential equations of the form

$$\begin{cases} \dot{x}(t) = F(x(t)), \quad t \geq 0, \\ x(0) = x_0 \in \mathbb{R}^d, \end{cases} \tag{1}$$

gives rise to a linear one by considering another state space. If the system is well-posed, we can associate a semiflow φ to the system (1) by taking $x(t) = \varphi(t, x_0)$ and derive the corresponding semigroup of Koopman operators, the so-called Koopman semigroup, as

$$(T(t)f)(x) := f(\varphi(t, x)). \tag{2}$$

The new state space is hence a linear space of functions defined on $\mathbb{R}^d$, typically a Banach space, and in this way we obtain a one-parameter semigroup $(T(t))_{t\geq 0}$ of linear operators. As it turns out, many properties of the solutions to (1) can be deduced from the appropriate properties of the associated Koopman semigroup or its generator (which is a derivation, i.e., $A(f \cdot g) = f \cdot Ag + Af \cdot g$).

In the working group we shall discuss possible choices of the new state space (concrete Banach spaces or just locally convex spaces with the appropriate topology) and possible properties of the solutions that can be read off from the corresponding Koopman semigroup or its generator. We shall also consider the case when the semiflow (dynamical system) is defined on a topological space X (i.e., not necessarily comes from an ODE). A particular emphasis will be given to case of non-(locally)-compact X, which is important if the semiflow comes from a PDE, and X is (a convex subset of) an infinite dimensional Banach space.

The aim of the working group is not much to solve concrete problems, but to find ones, and most of all we want to initiate discussion on this topic.