Unbounded convergences in vector and Banach lattices

VLADIMIR TROITSKY
Department of Mathematical and Statistical Sciences
University of Alberta (Canada)
troitsky@ualberta.ca

Let τ be a mode of convergence of nets in a vector lattice X. We define its "unbounded" counterpart as follows: $x_\alpha \xrightarrow{\mathrm{u}\tau} x$ if $|x_\alpha - x| \wedge u \xrightarrow{\tau} x$ for every $u \geq 0$. In my talk, I will present an overview of recent results by various authors on unbounded order (uo) convergence on vector lattices and unbounded norm (un) convergence on Banach lattices. For sequences in most function spaces, these convergences agree with convergence almost everywhere and with convergence in measure, respectively. Hence, one can think of uo and un convergences as generalizations of convergences everywhere and in measure, respectively. This allows one to extend various facts of measure theory and L_p-spaces to the much broader setting of function spaces and Banach lattices.

While uo convergence is not topological, un convergence is. We will discuss properties of un topology. In particular, un topology is metrizable iff the space has a quasi-interior point. I will also discuss extending un topology to the universal completion of the space.