Dimension of commuting varieties

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Let $C_r(\mathfrak{g})$ denote the set of all r-tuples of commuting elements of the Lie algebra $\mathfrak{g}$ of a linear algebraic group G, and $C_r(\mathcal{N})$ the subset of $C_r(\mathfrak{g})$ consisting of all r-tuples of commuting nilpotent elements. Both sets have natural structures of affine varieties. If $\mathfrak{g}$ is reductive, then $C_2(\mathfrak{g})$ is known to be irreducible and of dimension $\dim \mathfrak{g} + \operatorname{rank} \mathfrak{g}$, while $C_2(\mathcal{N})$ is equidimensional of dimension $\dim[G, G]$. On the other hand, for $r > 2$ these varieties are reducible, except for some small ranks of $\mathfrak{g}$, and very little is known about the irreducible components. We compute the dimension of $C_r(\mathfrak{g})$ and of $C_r(\mathcal{N})$ for sufficiently large r if $\mathfrak{g}$ is of type A or C and the characteristic of the ground field is neither 2 nor 3.

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