Minimal determinantal representations of bivariate polynomials

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It is known since Dixon's 1902 paper that every bivariate polynomial of degree n admits a determinantal representation with $n \times n$ symmetric matrices. However, the construction of such matrices is far from trivial and up to now there have been no efficient numerical algorithms, even if we do not insist on matrices being symmetric. We present a numerical construction of determinantal representations that returns $n \times n$ matrices for a square-free bivariate polynomial of degree n, which, with the exception of the symmetry, agrees with Dixon's result. For a non square-free polynomial one can combine it with a square-free factorization to obtain a representation of order n.

Our motivation is a novel numerical method for solving systems of bivariate polynomials as two-parameter eigenvalue problems. Symmetry is not important for this particular application.