Vector states on operator semigroups

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Let $\mathcal{S}$ be a multiplicative semigroup of bounded linear operators on a complex Hilbert space $\mathcal{H}$, and let Ω be the range of a vector state on $\mathcal{S}$ so that $\Omega = \{\langle S\xi, \xi \rangle : S \in \mathcal{S}\}$ for some fixed unit vector $\xi \in \mathcal{H}$. We study the structure of sets of cardinality two coming from irreducible semigroups $\mathcal{S}$. This leads us to sufficient conditions for reducibility and, in some cases, for the existence of common fixed points for $\mathcal{S}$. This is made possible by a thorough investigation of the structure of maximal families $\mathcal{F}$ of unit vectors in $\mathcal{H}$ with the property that there exists a fixed constant $\rho \in \mathbb{C}$ for which $\langle x, y \rangle = \rho$ for all distinct pairs x and y in $\mathcal{F}$.

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