Wiener's lemma along primes and other subsequences

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Inspired by ergodic theorems along subsequences, we study the validity of Wiener's lemma and extremal behavior of measures μ on the unit circle concerning their Fourier coefficients $\hat{\mu}(k_n)$ along subsequences (k_n) of $\mathbb{N}$, with focus on arithmetic subsequences such as polynomials, primes and polynomials of primes. We also discuss consequences for orbits of operators extending results of J. Goldstein and B. Nagy. As an application of the general results we shall prove among others the following facts which may seem surprising. Denote by p_n the n^{th} prime:

(1) If T is a (linear) contraction on a Hilbert space and $x \in H \setminus \{0\}$ is such that $|(T^{p_n}x|x)| \to \|x\|^2$ as $n \to \infty$, then x is an eigenvector of T to a unimodular eigenvalue.
(2) If T is a power bounded operator on a Banach space E and $x \in E \setminus \{0\}$ is such that $|\langle T^{p_n}x, x'\rangle| \to |\langle x, x'\rangle|$ as $n \to \infty$ for every $x' \in E'$, then x is an eigenvector of T to a unimodular eigenvalue.
(3) If T is a power bounded operator on a Banach space E with $T^{p_n} \to I$ in the weak operator topology, then $T = I$.

This is a joint work with TANJA EISNER (University of Leipzig).