On distributionally irregular vectors

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joint work with N.C. Bernandez, Jr., A. Bonila and A. Peris.

Definition

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Example

(Beauzamy) There exists $T \in B(H)$ such that each non-zero vector is irregular.

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(i) $\sup_n \|T^n\| = \infty$;
(ii) there exists $x \in X$ *such that* $\sup_n \|T^n x\| = \infty$;
(iii) there exists a residual subset $X_1 \subset X$ *consisting of vectors with unbounded orbits.*

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Then the set $\{x \in X : \inf_n \|T^n x\| = 0\}$ *is residual.*

Corollary

Let $T \in B(X)$. If the set of all irregular vectors is dense then it is residual.

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(iv) the set of all irregular vectors is residual;
(v) there exists a dense subspace $Y \subset X$ *consisting of irregular vectors (up to* 0*).*

Corollary

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Definition

Let $T \in B(X)$. A vector $x \in X$ is called distributionally irregular for T if there exist subsets $A, B \subset \mathbb{N}$ with $\overline{\text{dens}}\, A = 1$, $\overline{\text{dens}}\, B = 1$,

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(iii) for every k there exists $y_k \in X$, $\|y_k\| = 1$ and n_k such that

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(iv) there exists $\varepsilon > 0$, such that for every k there exists $y_k \in X$, $\|y_k\| = 1$ and n_k such that

$$\mathrm{card}\,\{n \le n_k : \|T^n y_k\| > k\} \ge \varepsilon n_k.$$

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Let $T_1 \in B(X)$. Suppose that the set of all vectors with orbits distributionally unbounded away from 0 is dense.

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Corollary

Let $T \in B(X)$. If the set of all distributionally irregular vectors is dense then it is residual.

Definition

Let M be a metric space. A continuous mapping $f : M \to M$ is called distributionally chaotic if there exists an uncountable set $\Gamma \subset M$ and $\varepsilon > 0$ such that for all $x, y \in \Gamma$, $x \neq y$ there exist subsets $A, B \subset \mathbb{N}$ with upper density 1 such that

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(i) there exists a vector $x \in X$ *with distributionally unbounded orbit;*

(ii) there exists a dense linear manifold consisting (up to 0) from distributionally irregular vectors.