Open linear maps and geometry of the numerical range

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The maximum-entropy principle goes back to Boltzmann's pioneering ideas in statistical mechanics and von Neumann has generalized the maximum-entropy principle to quantum systems where it plays a major role in quantum statistical mechanics and quantum state reconstruction. Given the expected values of a set of observables, the *maximum-entropy inference* solves the inverse problem of selecting a quantum state which satisfies the expected value constraints by maximizing an entropic functional.

A. Knauf and myself have observed that the maximum-entropy inference can be discontinuous [1]. This can never happen for commutative observables. I show in [2] that the continuity of a family of constrained optimization problems is equivalent to the *openness* of the projection to the constraint sets. For a family of parallel linear constraints on a convex body I provide sufficient conditions for the openness. They solve for certain size-three block-diagonal matrices the continuity problem of the maximum-entropy inference. A discussion for arbitrary size-three matrices is still in its initial stages.

The set of expected value pairs of two observables equals the *numerical range* [3]. From the perspective of the Grassmannian of two-dimensional planes, spanned by pairs of observables, a discontinuous inference appears where non-exposed points of the numerical range disappear [1]. This holds for certain size-three block-diagonal matrices and I would like to discuss this problem more broadly by invoking the continuous—open equivalence and geometric results about the numerical range [3].

References

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[3] L. Rodman, I. M. Spitkovsky, 3×3-*Matrices with a Flat Portion on the Boundary of the Numerical Range*, Linear Algebra Appl. 397 (2005), 193–207.