On 5×5 golden magic matrices and 5×5 *Stifelsche Quadrate*

GEORGE P. H. STYAN
Department of Mathematics and Statistics
McGill University (Canada)
geostyan@gmail.com

We comment on some properties of 5×5 golden magic matrices and 5×5 *Stifelsche Quadrate.* Our "golden magic matrices" $\mathbf{M}$ are 5×5 fully-magic matrices where $\mathbf{M}^2$ is a Toeplitz-circulant with top row (p, q, r, r, q) and eigenvalues $a + \Phi b$; here $\Phi = (1 + \sqrt{5})/2$ is the Golden Ratio and a, b are rational numbers. For example, we find that the column-flipped Agrippa–Paracelsus classic fully-magic matrix $\mathbf{A}$ and the fully-magic Latin-square Hankel-circulant $\mathbf{H}$ are golden

$$\mathbf{A} = \begin{pmatrix} 3 & 20 & 7 & 24 & 11 \\ 16 & 8 & 25 & 12 & 4 \\ 9 & 21 & 13 & 5 & 17 \\ 22 & 14 & 1 & 18 & 10 \\ 15 & 2 & 19 & 6 & 23 \end{pmatrix}, \qquad \mathbf{H} = \begin{pmatrix} 0 & 3 & 1 & 4 & 2 \\ 3 & 1 & 4 & 2 & 0 \\ 1 & 4 & 2 & 0 & 3 \\ 4 & 2 & 0 & 3 & 1 \\ 2 & 0 & 3 & 1 & 4 \end{pmatrix}. \tag{1}$$

We define 5×5 *Stifelsche Quadrate* as 5×5 fully-magic matrices where the 3×3 inner centre submatrix is also fully-magic, for example

$$\mathbf{S} = \begin{pmatrix} 3 & 18 & 21 & 22 & 1 \\ 24 & 16 & 11 & 12 & 2 \\ 7 & 9 & 13 & 17 & 19 \\ 6 & 14 & 15 & 10 & 20 \\ 25 & 8 & 5 & 4 & 23 \end{pmatrix}. \tag{2}$$

The 5×5 magic square defined by $\mathbf{S}$ was chosen recently for a German postage stamp in honour of the mathematician and Augustinian monk Michael Stifel (1487–1567). The matrix $\mathbf{S}$ with magic sum $m = 65$ is not golden but has a nilpotent property in common with the golden matrices $\mathbf{A}$ and $\mathbf{H}$: we find that $\mathbf{N}_{\mathbf{S}}^2 = \mathbf{0}$ where $\mathbf{N}_{\mathbf{S}} = \mathbf{S} + \mathbf{SF} - 2m\bar{\mathbf{E}}$, with $\mathbf{F}$ the flip matrix and $\bar{\mathbf{E}}$ the 5×5 matrix with every entry equal to $1/5$. Moreover, $\mathbf{N}_{\mathbf{A}}^2 = \mathbf{N}_{\mathbf{H}}^2 = \mathbf{0}$.

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