

Local and global liftings of analytic families of idempotents in Banach algebras

ENDRE MAKAI, JR.

*Alfréd Rényi Mathematical Institute
Hungarian Academy of Sciences (Hungary)*
makai.endre@renyi.mta.hu

Generalizing results of our earlier paper, we investigate the following question. Let $\pi(\lambda) : A \rightarrow B$ be an analytic family of surjective homomorphisms between two Banach algebras, and $q(\lambda)$ an analytic family of idempotents in B . We want to find an analytic family $p(\lambda)$ of idempotents in A , lifting $q(\lambda)$, i.e., such that $\pi(\lambda)p(\lambda) = q(\lambda)$, under hypotheses of the type that the elements of $\text{Ker } \pi(\lambda)$ have small spectra. For spectra which do not disconnect \mathbb{C} we obtain a local lifting theorem. For real analytic families of surjective $*$ -homomorphisms (for continuous involutions) and self-adjoint idempotents we obtain a local lifting theorem, for totally disconnected spectra. We obtain a global lifting theorem if the spectra of the elements in $\text{Ker } \pi(\lambda)$ are $\{0\}$, both in the analytic case, and, for $*$ -algebras (with continuous involutions) and self-adjoint idempotents, in the real analytic case. Here even an at most countably infinite set of mutually orthogonal analytic families of idempotents can be lifted to mutually orthogonal analytic families of idempotents. In the proofs, spectral theory is combined with complex analysis and general topology, and even a connection with potential theory is mentioned.

This is a joint work with B.AUPETIT (Université Laval), M. MBEKHTA (Université des Sciences et Technologies de Lille) and J. ZEMÁNEK (Polish Academy of Sciences).