(Exactly) Paratransitive algebras of linear transformations

LEO LIVSHITS
Department of Mathematics and Statistics
Colby College (USA)
llivshi@colby.edu

We study a natural weakening – which we refer to as *paratransitivity* – of the well-known notion of transitivity of an algebra $\mathcal{A}$ of linear transformations acting on a finite-dimensional vector space $\mathcal{V}$. Given positive integers k and m, we shall say that such an algebra $\mathcal{A}$ is (k, m)-transitive if for every pair of subspaces $\mathcal{W}_1$ and $\mathcal{W}_2$ of $\mathcal{V}$ of dimensions k and m respectively, we have $\mathcal{A}\mathcal{W}_1 \cap \mathcal{W}_2 \neq \{0\}$. We consider the structure of minimal (k, m)-transitive algebras and explore the connection of this notion to a measure of largeness for invariant subspaces of $\mathcal{A}$.

We will also introduce a related, and more restrictive notion of an "*exact paratransitivity*". An algebra $\mathcal{A}$ is exactly (k, m)-transitive if the image (under the algebra) of every k-dimensional space has co-dimension $m - 1$. Again, in this case we are able to classify such algebras in a number of setting.

To discern the structure of the paratransitive algebras we develop a spatial implementation of "Wedderburn's Principal Theorem", which may be of interest in its own right: if a subalgebra $\mathcal{A}$ of $\mathcal{L}(\mathcal{V})$ is represented by block-upper-triangular matrices with respect to a maximal chain of its invariant subspaces, then after an application of a block-upper-triangular similarity, the block-diagonal matrices in the resulting algebra comprise its Wedderburn factor. In other words, we show that, up to a block-upper-triangular similarity, $\mathcal{A}$ is a linear direct sum of an algebra of block-diagonal matrices and an algebra of strictly block-upper-triangular matrices.

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