Sensitivity analysis for perfect state transfer in quantum walks

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Suppose that G is a graph on vertices $1, \ldots, n$ with adjacency matrix A, and for each $t \geq 0$, let $U(t) = exp(itA)$, where exp denotes the matrix exponential. Fixing an index k between 1 and n, it is straightforward to determine that $\sum_{j=1}^{n} |u_{k,j}(t)|^2 = 1$; the vectors $\left[\begin{array}{ccc} |u_{k,1}(t)|^2 & \ldots & |u_{k,n}(t)|^2 \end{array} \right]$, $t \geq 0$ can be thought of as a *continuous time quantum walk* on G, starting from vertex k. That is, $|u_{k,j}(t)|^2$ represents the probability that a quantum walk on G starting from vertex k arrives at vertex j at time t.

The quantities $|u_{k,j}(t)|^2, j = 1, \ldots, n$ are of interest in quantum physics. For a network (represented by G) of interacting quantum states, a state is input at vertex k, and after time t has elapsed, the state is read out at vertex j. The quantity $|u_{k,j}(t)|^2$, which is the known as the *fidelity*, measures the similarity between the original state input at k, and the state read out at j. In particular, if $|u_{k,j}(t_0)|^2 = 1$ for some $t_0 > 0$, then we say there is *perfect state transfer* from k to j at time t_0. The last decade has seen a good deal of interest in perfect state transfer, in part because it serves as a model for the transfer of information in a quantum computer.

In the setting of perfect state transfer we consider, in this talk, the sensitivity of the fidelity with respect to both the readout time t_0, and the intensity of the interactions between spins. Using techniques from matrix analysis, we derive expressions for the derivatives of the fidelity with respect to both types of quantities. The results may help to inform the design of spin networks that not only exhibit perfect state transfer but also offer some forgiveness to errors in readout time and/or spin interactions.