Hawaiian volcanoes and complete polynomial bounds

JOHN HOLBROOK Department of Mathematics and Statistics University of Guelph (Canada) jholbroo@uoguelph.ca

The recent paper [1] completes work begun some years ago with Frank Gilfeather at the Maui High Performance Computing Center (MHPCC, located on the Hawaiian island of Maui). It combines that work with important new ideas due to Michel Crouzeix. We explain how the problem treated in [1] developed from a highly influential 1970 paper by Paul Halmos, which drew attention to ten research problems about Hilbert space operators. Among the most stimulating was the following: find an intrinsic property of an operator *T* that holds iff *T* is similar to a contraction *C*. Halmos proposed that such a property might be: $K(T) < \infty$, where K(T) is the so-called polynomial bound of *T*, ie the supremum of ||p(T)|| over polynomials *p* mapping the unit disc into itself.

Many important tools were developed in response to this problem, notably by Arveson, Paulsen, Bourgain, Pisier, and Davidson. Pisier finally (c.1995) showed that the Halmos criterion must be strengthened. We'll give an account of these developments (suitable for a general mathematical audience) leading up to the related puzzle resolved in our joint work [1].

This talk will be based on a joint work with M. CROUZEIX (Université de Rennes) and F. GILFEATHER (University of New Mexico).

References

[1] M. Crouzeix, F. Gilfeather, J. Holbrook, *Polynomial bounds for small matrices*, Linear and Multilinear Algebra, DOI: 10.1080/03081087.2013.777439.