Invited ILAS Lecture

Pólya permanent problem: 100 years after

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Two important functions in matrix theory, determinant and permanent, look very similar:

$$\det A = \sum_{\sigma \in S_n} (-1)^{\sigma} a_{1\sigma(1)} \cdots a_{n\sigma(n)} \quad \text{and} \quad \operatorname{per} A = \sum_{\sigma \in S_n} a_{1\sigma(1)} \cdots a_{n\sigma(n)}$$

here $A = (a_{ij}) \in M_n(\mathbb{F})$ is an $n \times n$ matrix and S_n denotes the set of all permutations of the set $\{1, \ldots, n\}$.

While the computation of the determinant can be done in a polynomial time, it is still an open question, if there are such algorithms to compute the permanent. Due to this reason, starting from the work by Pólya, 1913, different approaches to convert the permanent into the determinant were under the intensive investigation.

The lecture will contain the exposition of this theory during the past 100 years including our recent research results.

This talk is based on a series of joint works with M. BUDREVICH (Lomonosov Moscow State University), G. DOLINAR (University of Ljubljana), B. KUZMA (University of Primorska) and M. OREL (University of Primorska).

Invited talks at LAW

Hawaiian volcanoes and complete polynomial bounds

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The recent paper [1] completes work begun some years ago with Frank Gilfeather at the Maui High Performance Computing Center (MHPCC, located on the Hawaiian island of Maui). It combines that work with important new ideas due to Michel Crouzeix. We explain how the problem treated in [1] developed from a highly influential 1970 paper by Paul Halmos, which drew attention to ten research problems about Hilbert space operators. Among the most stimulating was the following: find an intrinsic property of an operator T that holds iff T is similar to a contraction C. Halmos proposed that such a property might be: $K(T) < \infty$, where $K(T)$ is the so-called polynomial bound of T, ie the supremum of $\|p(T)\|$ over polynomials p mapping the unit disc into itself.

Many important tools were developed in response to this problem, notably by Arveson, Paulsen, Bourgain, Pisier, and Davidson. Pisier finally (c.1995) showed that the Halmos criterion must be strengthened. We'll give an account of these developments (suitable for a general mathematical audience) leading up to the related puzzle resolved in our joint work [1].

This talk will be based on a joint work with M. CROUZEIX (Université de Rennes) and F. GILFEATHER (University of New Mexico).

References

[1] M. Crouzeix, F. Gilfeather, J. Holbrook, *Polynomial bounds for small matrices*, Linear and Multilinear Algebra, DOI: 10.1080/03081087.2013.777439.

Patterned matrices with explicit trace vector and some consequences

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Let A be an $n \times n$ complex matrix. R. Pereira has proved that there is an $n-$vector v such that $v^* p(A) v = trace(p(A)/n)$, for all polynomials $p(A)$, and he calls such a vector v a trace vector for A. While finding a trace vector for an arbitrary A is difficult, there are many classes of matrices for which a trace vector is easily found. We present several examples of this and its consequences. In particular, a patterned matrix playing a type of universal role in the nonnegative inverse eigenvalue problem has this property, and we deduce new results on the sign patterns of the coefficients of related power series.

This is a joint work with R. LOEWY (Technion) and H. ŠMIGOC (University College Dublin).

(Exactly) Paratransitive algebras of linear transformations

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We study a natural weakening – which we refer to as *paratransitivity* – of the well-known notion of transitivity of an algebra $\mathcal{A}$ of linear transformations acting on a finite-dimensional vector space $\mathcal{V}$. Given positive integers k and m, we shall say that such an algebra $\mathcal{A}$ is (k, m)-transitive if for every pair of subspaces $\mathcal{W}_1$ and $\mathcal{W}_2$ of $\mathcal{V}$ of dimensions k and m respectively, we have $\mathcal{A}\mathcal{W}_1 \cap \mathcal{W}_2 \neq \{0\}$. We consider the structure of minimal (k, m)-transitive algebras and explore the connection of this notion to a measure of largeness for invariant subspaces of $\mathcal{A}$.

We will also introduce a related, and more restrictive notion of an "*exact paratransitivity*". An algebra $\mathcal{A}$ is exactly (k, m)-transitive if the image (under the algebra) of every k-dimensional space has co-dimension $m - 1$. Again, in this case we are able to classify such algebras in a number of setting.

To discern the structure of the paratransitive algebras we develop a spatial implementation of "Wedderburn's Principal Theorem", which may be of interest in its own right: if a subalgebra $\mathcal{A}$ of $\mathcal{L}(\mathcal{V})$ is represented by block-upper-triangular matrices with respect to a maximal chain of its invariant subspaces, then after an application of a block-upper-triangular similarity, the block-diagonal matrices in the resulting algebra comprise its Wedderburn factor. In other words, we show that, up to a block-upper-triangular similarity, $\mathcal{A}$ is a linear direct sum of an algebra of block-diagonal matrices and an algebra of strictly block-upper-triangular matrices.

This is a joint work with G. MACDONALD (University of Prince Edward Island), L.W. MARCOUX (University of Waterloo) and H. RADJAVI (University of Waterloo).

Extending semigroups of partial isometries

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A partial isometry V acting on a Hilbert space $\mathcal{H}$ is a continuous linear map which satisfies $(V^*V)^2 = V^*V$.

Let $\mathcal{S}$ be a semigroup of partial isometries acting on a complex, infinite-dimensional, separable Hilbert space. In this paper we seek criteria which will guarantee that the selfadjoint semigroup $\mathcal{T}$ generated by $\mathcal{S}$ consists of partial isometries as well. Amongst other things, we show that this is the case when the set $\mathcal{Q}(\mathcal{S})$ of final projections of elements of $\mathcal{S}$ generates an abelian von Neumann algebra of uniform finite multiplicity.

This is joint work with J. BERNIK (University of Ljubljana), A. POPOV (University of Waterloo) and H. RADJAVI (University of Waterloo).

On semitransitivity

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We say that a collection C of complex $n \times n$ matrices is semitransitive, or, more precisely, acts semitransitively on the underlying n-dimensional vector space V, if for every pair of nonzero vectors x, y in V there is an element A of C such that either $Ax = y$ or $Ay = x$. The notion coincides with the notion of transitivity for groups of matrices, but not in general. Topological version of the notion can is defined in the obvious way. Semitransitivity was introduced in 2005 by H. Rosenthal and V. Troitsky who first studied it in the context of WOT-closed algebras of Hilbert space operators. It was later studied in finite and infinite dimensional settings by many authors - including a working group at LAW. A good deal of results were obtained, sometimes in line with initial conjectures but quite often not.

This will be a survey talk of some interesting results and tools used in the area. I will also discuss recent work, joint with J. Bernik, in which we relate the notion of semitransitivity to the study of prehomogeneous vector spaces.

On distributionally irregular vectors

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Let T be a bounded linear operator acting on a Banach space X. A vector $x \in X$ is called irregular if $\sup \|T^n x\| = \infty$ and $\inf \|T^n x\| = 0$. The notion was introduced by Beauzamy and is closely connected with hypercyclicity of vectors.

We consider a related notion of distributionally irregular vectors. A vector $x \in X$ is called distributionally irregular if there exist subsets A, B of natural numbers with upper density 1 such that $\lim_{n \in A} \|T^n x\| = \infty$ and $\lim_{n \in B} \|T^n x\| = 0$. Both irregular and distributionally irregular vectors were studied in the context of dynamical systems (under the names of Li-Yorke chaos and distributional chaos, respectively).

This is a joint work with N. C. BERNANDES JR. (Universidade Federal do Rio de Janeiro), A. BONILLA (Universidad de la Laguna) and A. PERIS (Universitat Politècnica de València).

Simultaneous versions of Wielandt's positivity theorem

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The classical Wielandt Theorem is about "positivization" of a matrix: If an indecomposable matrix A and and its modulus $|A|$ have the same spectral radius, then, after a diagonal similarity, A is just a scalar multiple of $|A|$. Here $|A|$ is the matrix whose entries are the moduli of those of A; and "indecomposable" means that no nontrivial subset of the basic vectors spans an invariant subspace for (the operator whose matrix relative to this basis is) A. In joint work with Gordon Macdonald we present extensions of this result to certain semigroups of operators in finite and infinite dimensions.

This is a joint work with G. MACDONALD (University of Prince Edward Island).

Numerical ranges of quaternion matrices

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In the talk, results concerning numerical ranges and joint numerical ranges of matrices are reviewed. The matrices are assumed to have entries in the skew field of real quaternions. Convexity (or non-convexity) properties, as well as connections between geometric properties of numerical ranges and joint numerical ranges and algebraic properties of the matrix involved, are explored. Together with the classical concepts of numerical ranges, we consider also (joint) numerical ranges with respect to general antiautomorphisms of real quaternions. Open problems will be formulated.

The semigroup generated by a unitary orbit

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The unitary orbit of a complex $n \times n$ matrix A is the set $\{U^*AU \mid U \text{ unitary}\}$. We investigate the structure of the multiplicative semigroup generated by the unitary orbit of a matrix.

This is a joint work with H. RADJAVI (University of Waterloo).

Multinorms and Banach lattices

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For every $n \in \mathbb{N}$, let $\|\cdot\|_n$ be a norm on X^n, where X is a fixed vector space. The resulting sequence of norms is called a *multinorm* provided that is satisfies certain natural compatibility axioms. Multinormed spaces were introduced by Dales and Polyakov. They were investigated by Pisier and others in the language of tensor norms on $c_0 \otimes X$. Multinormed spaces can be identified with subspaces of Banach lattices. In this talk, we will discuss connections between multinormed spaces and

Banach lattices. We will also discuss a more general concept of a p-multinorm, $1 \leq p \leq \infty$.

This is a joint work with G. DALES (Lancaster University) and N. LAUSTSEN (Lancaster University).

Invited talks at IWMS

A new difference-based weighted mixed Liu estimator in partially linear models

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In this paper, a generalized difference-based estimator is introduced for the vector parameter β in partially linear model when the errors are correlated. A generalized difference-based Liu estimator is defined for the vector parameter β . Under the linear stochastic constraint $r = R\beta + e$, we introduce a new generalized-based weighted mixed Liu estimator. The efficiency properties of the difference-based weighted mixed regression method is analyzed.

This is a joint work with E. AKDENIZ DURAN (Istanbul Medeniyet University).

Change detection in polarimetric SAR images using complex Wishart distributed matrices

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In surveillance it is important to be able to detect natural or man-made changes e.g. based on sequences of satellite or air borne images of the same area taken at different times, The mapping capability of synthetic aperture radar (SAR) is independent of e.g. cloud cover, and thus this technology holds a strong potential for change detection studies in remote sensing. In polarimetric synthetic aperture radar we measure the amplitude and phase of backscattered signals in four combinations of the linear horizontal and vertical receive and transmit polarizations. The-se signals form a complex scattering matrix, and after suitable preprocessing the outcome at each picture element (pixel) may be represented as a 3 by 3 Hermitian matrix following a complex Wishart distribution.

One approach to solving the change detection problem based on SAR images is therefore to apply suitable statistical tests in the complex Wishart distribution. We propose a set-up for a systematic solution to the (practical) problems using the likelihood ratio test statistics. We show some examples based on a time series of images with 1024 by 1024 pixels.

This talk reports joint work with A. Aasbjerg Nielsen (Technical University of Denmark) and H. Skriver (Technical University of Denmark).

Analyzing Markov chains using Kronecker products

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Kronecker products are used to define the underlying Markov chain (MC) in various modeling formalisms, including compositional Markovian models, hierarchical Markovian models, and stochastic process algebras. Although the Kronecker representation does not provide a solution to the storage problem of state probability vectors associated with the model, it enables the storage of the underlying state transition matrix compactly, thereby facilitating the analysis of multi-dimensional models that are an order of magnitude larger than those that can be analyzed with conventional sparse matrix techniques on the same platform due to memory limitations. In the Kronecker based approach, the generator matrix underlying the MC is represented using Kronecker products of smaller matrices and is never explicitly generated. The implementation of transient and steady–state solvers rests on this compact Kronecker representation, thanks to the existence of an efficient vector–Kronecker product multiplication algorithm known as the shuffle algorithm. Here, we take a vector–matrix approach and discuss recent results related to the analysis of MCs based on Kronecker products independently from modeling formalisms.

Riesz probability distribution on symmetric matrices and extensions of the Olkin-Rubin characterization

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We introduce the Riesz probability distribution on the cone of positive symmetric matrices as a generalization of the Wishart distribution. We then define some related distributions and established some properties. Versions of the Olkin-Rubin theorem without invariance of the "quotient" are given.

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Sensitivity analysis for perfect state transfer in quantum walks

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Suppose that G is a graph on vertices $1, \dots, n$ with adjacency matrix A, and for each $t \geq 0$, let $U(t) = exp(itA)$, where exp denotes the matrix exponential. Fixing an index k between 1 and n, it is straightforward to determine that $\sum_{j=1}^{n} |u_{k,j}(t)|^2 = 1$; the vectors $\left[\ |u_{k,1}(t)|^2 \quad \dots \quad |u_{k,n}(t)|^2 \ \right]$, $t \geq 0$ can be thought of as a *continuous time quantum walk* on G, starting from vertex k. That is, $|u_{k,j}(t)|^2$ represents the probability that a quantum walk on G starting from vertex k arrives at vertex j at time t.

The quantities $|u_{k,j}(t)|^2, j = 1, \dots, n$ are of interest in quantum physics. For a network (represented by G) of interacting quantum states, a state is input at vertex k, and after time t has elapsed, the state is read out at vertex j. The quantity $|u_{k,j}(t)|^2$, which is the known as the *fidelity*, measures the similarity between the original state input at k, and the state read out at j. In particular, if $|u_{k,j}(t_0)|^2 = 1$ for some $t_0 > 0$, then we say there is *perfect state transfer* from k to j at time t_0. The last decade has seen a good deal of interest in perfect state transfer, in part because it serves as a model for the transfer of information in a quantum computer.

In the setting of perfect state transfer we consider, in this talk, the sensitivity of the fidelity with respect to both the readout time t_0, and the intensity of the interactions between spins. Using techniques from matrix analysis, we derive expressions for the derivatives of the fidelity with respect to both types of quantities. The results may help to inform the design of spin networks that not only exhibit perfect state transfer but also offer some forgiveness to errors in readout time and/or spin interactions.

Life distributions in survival analysis and reliability: Structure of nonparametric, semiparametric and parametric families

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Keywords: survival analysis, reliability theory, log concave densities, hazard rates, stochastic orders, nonparametric families, semiparametric families

In this talk I will discuss some of the characteristics of life distributions that arise in survival analysis and reliability theory. Alternative definitions of a distribution are discussed and then related to a variety of stochastic orders: hazard rate order, likelihood ratio order, convex order, Lorenz order. Nonparametric families, in particular log concave densities, completely monotone distributions , increasing hazard rate families, new-better-than-used families and bathtub hazard rate families are analyzed. A taxonomy for semiparametric families is presented and the effect of introducing parameters on various stochastic orders is shown. Finally, the introduction of covariate models in these families is developed.

On 5×5 golden magic matrices and 5×5 *Stifelsche Quadrate*

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We comment on some properties of 5×5 golden magic matrices and 5×5 *Stifelsche Quadrate.* Our "golden magic matrices" $\mathbf{M}$ are 5×5 fully-magic matrices where $\mathbf{M}^2$ is a Toeplitz-circulant with top row (p, q, r, r, q) and eigenvalues $a + \Phi b$; here $\Phi = (1+\sqrt{5})/2$ is the Golden Ratio and a, b are rational numbers. For example, we find that the column-flipped Agrippa–Paracelsus classic fully-magic matrix $\mathbf{A}$ and the fully-magic Latin-square Hankel-circulant $\mathbf{H}$ are golden

$$\mathbf{A} = \begin{pmatrix} 3 & 20 & 7 & 24 & 11 \\ 16 & 8 & 25 & 12 & 4 \\ 9 & 21 & 13 & 5 & 17 \\ 22 & 14 & 1 & 18 & 10 \\ 15 & 2 & 19 & 6 & 23 \end{pmatrix}, \qquad \mathbf{H} = \begin{pmatrix} 0 & 3 & 1 & 4 & 2 \\ 3 & 1 & 4 & 2 & 0 \\ 1 & 4 & 2 & 0 & 3 \\ 4 & 2 & 0 & 3 & 1 \\ 2 & 0 & 3 & 1 & 4 \end{pmatrix}. \tag{1}$$

We define 5×5 *Stifelsche Quadrate* as 5×5 fully-magic matrices where the 3×3 inner centre submatrix is also fully-magic, for example

$$\mathbf{S} = \begin{pmatrix} 3 & 18 & 21 & 22 & 1 \\ 24 & 16 & 11 & 12 & 2 \\ 7 & 9 & 13 & 17 & 19 \\ 6 & 14 & 15 & 10 & 20 \\ 25 & 8 & 5 & 4 & 23 \end{pmatrix}. \tag{2}$$

The 5×5 magic square defined by $\mathbf{S}$ was chosen recently for a German postage stamp in honour of the mathematician and Augustinian monk Michael Stifel (1487–1567). The matrix $\mathbf{S}$ with magic sum $m = 65$ is not golden but has a nilpotent property in common with the golden matrices $\mathbf{A}$ and $\mathbf{H}$: we find that $\mathbf{N}_{\mathbf{S}}^2 = \mathbf{0}$ where $\mathbf{N}_{\mathbf{S}} = \mathbf{S} + \mathbf{SF} - 2m\bar{\mathbf{E}}$, with $\mathbf{F}$ the flip matrix and $\bar{\mathbf{E}}$ the 5×5 matrix with every entry equal to $1/5$. Moreover, $\mathbf{N}_{\mathbf{A}}^2 = \mathbf{N}_{\mathbf{H}}^2 = \mathbf{0}$.

Joint research with M. A. AMELA (General Pico).

One-parameter semigroups of endomorphisms of a symmetric cone

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Let C be a closed cone in a Euclidean space V. A linear map $A\colon V \to V$ is called an endomorphism of the cone C or a positive map if $A(C) \subseteq C$. Let $\{e^{tA}; t \geq 0\}$ be a one-parameter semigroup of endomorphisms of the cone C. If C is polyhedral, then it is well-known that the generator A of the semigroup can be written as a sum of an endomorphism of C and a generator of one-parameter group of automorphisms of C. It is known that such a decomposition does not exist in general, but it is not known whether it exists if the cone C is symmetric. i.e. homogeneous and self-dual. We answer this question negatively. Explicitely, for each symmetric cone C of rank at least 3 we find a generator of a one-parameter semigroup of endomorphisms of C that cannot be written as a sum of an endomorphism of C and a generator of a one-parameter group of automorphisms of C. The work is motivated by the study of affine processes on symmetric cones.

This is a joint work with B. KUZMA (University of Primorska), M. OMLADIČ (University of Ljubljana) and J. TEICHMANN (ETH Zürich).

The role of coupling and the deviation matrix in calculating the value of capacity for queueing systems

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In queues with finite capacity C, customers are lost when they arrive to find C customers already present. Assuming that each arriving customer brings a certain amount of revenue, we are interested in calculating the value of an extra unit of capacity by deriving the expected amount of extra revenue earned over a finite time horizon $[0, T]$.

There are different ways of approaching this problem. One involves the derivation of Markov renewal equations by conditioning on the first instance at which the state of the queue changes. A second involves an elegant coupling argument. We shall describe both of these approaches and the role that the deviation matrix of the Markov chain plays in the analysis.

This is joint work with P. BRAUNSTEINS (University of Melbourne) and S. HAUTPHENNE (University of Melbourne).

Expansion formulas for inertias of quadratic matrix-valued functions with applications

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I shall introduce how to establish expansion formulas for calculating inertias of quadratic matrix-valued functions, and present their applications in characterizing mathematical and statistical properties of estimations in regression analysis.

Semi Markov migration process in a stochastic environment in credit risk

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In the present the idea of stochastic Market environment comes into play to express the changes in general economy, which affects any industry in small or great amounts of turbulence. We model the evolution of the Market among its possible -states as an F-inhomogeneous semi-Markov process. This idea leads us to modeling the migration process of defaultable bonds as different F-inhomogeneous semi-Markov process. The survival probabilities of a defaultable bond in every credit grade are found. The asymptotic behaviour of the survival probabilities is established under certain conditions. Also, it is proved under what conditions the convergence is geometrically fast. The stochastic foundation of the general stochastic discrete-time Market is provided, by proving that the market is viable, if and only if, there exists an equivalent martingale measure, from which we construct the forward probability measure and under which the discounted default free bond price process for all possible states of the Market is a martingale. The term structure of credit spread and the change of real-world probability measure to forward probability measure are studied. In the form of a Theorem it is proved that under certain conditions, changing the real probability measure to a forward probability measure, does not affect the inhomogeneous semi Markov process modeling the migration of defaultable bonds. That is, it is proved that it only changes the basic parameters and we provide a relation among the transition probabilities under the two measures. Finally, parameter estimation and calibration of the inhomogeneous semi-Markov chain in stochastic environment is being provided.

Contributed talks at LAW

Simultaneously self-adjoint sets of matrices

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The article addresses the following two questions:

(1) Consider a set of matrices $\mathcal{M} \subset \mathbb{C}^{d\times d}$. When are all the elements of $\mathcal{M}$ simultaneously equivalent to hermitian matrices under the natural action of $\mathrm{GL}_d(\mathbb{C}) \times \mathrm{GL}_d(\mathbb{C})$? In other words, when do there exist $A, B \in \mathrm{GL}_d(\mathbb{C})$ such that ANB is hermitian for all $N \in \mathcal{M}$?
(2) Assume that the answer to (1) is positive. Is there an element in $\mathcal{M}$ that is equivalent (under this simultaneous equivalence) to a positive definite matrix? In other words, given a set of hermitian $d \times d$ matrices, when do these matrices admit a positive definite linear combination?

We connect three approaches to study the above questions:

- linear algebra (simultaneous linear transformations of a set of matrices to symmetric or hermitian form)
- algebraic geometry (cubic curves, surfaces and hypersurfaces as zero loci of determinants)
- semidefinite programming (linear matrix inequality (LMI) representations)

Computationally both questions are straightforward. Question (1) reduces to a system of linear equations over $\mathbb{R}$. Question (2) is solved by semidefinite programming (at least for small d).

The study of simultaneous classification of matrices is equivalent to the geometric problem of determinantal representations. To the set $\mathcal{M}$ with a basis $\{M_0, \ldots, M_n\}$ we assign the matrix $M = x_0M_0 + x_1M_1 + \ldots + x_nM_n$, a *determinantal representation* of the homogeneous polynomial $\det M$. Definiteness of $\mathcal{M}$ imposes strong constraints on the determinant. For example, LMI hermitian representations induce hyperbolic polynomials / hyperbolicity cones.

For a generic set $\mathcal{M}$ with a chosen basis the corresponding determinant defines a smooth hypersurface of degree d in $\mathbb{P}^n$.

Our main interest is $d = 3$. In this case Question (1) reduces to testing four dimensional subsets in $\mathcal{M}$, where we use the theory of determinantal representations of cubic surfaces in $\mathbb{P}^3$. The reduction uses the theory of determinantal representations of cubic curves in $\mathbb{P}^2$.

We investigate the connection between definiteness and the existence of self-orthogonal vectors.

This is a joint work with T. KOŠIR (University of Ljubljana).

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Matrix convertibility over finite field

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Matrix A of order n is convertible if it is possible to multiply some of its elements by minus ones in such a way that for the obtained matrix A' the permanent of A is equal to the determinant of A'. The notion of convertibility was defined by Pólya as an alternative way to compute permanent function. Convertibility is well studied only for the set of (0,1)-matrices.

Another way to compute permanent function of integer-valued matrix is to compute $\mathrm{per}(A) \pmod{k}$ for different integer k and apply Chinese remainder theorem for obtained values. If k is a prime number then computing $\mathrm{per}(A) \pmod{k}$ is equivalent to computing of the permanent function over finite field with k elements. This is the reason to study the convertibility for matrices over finite fields.

In the talk some properties of convertibility for matrices over finite fields will be discussed. Among the properties under discussion there are some results for (0,1)-matrices in the case of finite fields and sufficient condition of convertibility over finite field.

An elementary proof of Wigner's theorem on quantum mechanical symmetry transformations

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Let $\mathcal{H}$ be a complex Hilbert space and $\mathcal{P}_1 = \mathcal{P}_1(\mathcal{H})$ denote the set of *rank-one and self-adjoint projections* on $\mathcal{H}$, i.e.

$$\mathcal{P}_1 = \{\mathbf{P}[\vec{v}] \colon \vec{v} \in \mathcal{H}, \|\vec{v}\| = 1\}$$

where $\mathbf{P}[\vec{v}]$ refers to the projection with precise range $\mathbb{C} \cdot \vec{v}$ (the notations $|\vec{v}\rangle\langle\vec{v}|$ or $\vec{v} \otimes \vec{v}$ are also favourable versions for $\mathbf{P}[\vec{v}]$). We note that the so-called *unit rays* (or *pure states*) of $\mathcal{H}$ and the *one-dimensional subspaces* of $\mathcal{H}$ can be identified with $\mathcal{P}_1$ in a very natural way.

The *transition probability* between two elements $\mathbf{P}[\vec{v}]$ and $\mathbf{P}[\vec{w}]$ is the quantity

$$\operatorname{tr} \mathbf{P}[\vec{v}]\mathbf{P}[\vec{w}] = |\langle \vec{v}, \vec{w} \rangle|^2.$$

Wigner's theorem on quantum symmetry transformations is very important in quantum mechanics which belongs to the mathematical foundation of the subject. It states that a mapping on $\mathcal{P}_1$ which preserves the transition probability is induced by a linear or antilinear isometry of the underlying Hilbert space $\mathcal{H}$. We note that there are some other equivalent formulations of the above theorem.

The classical version (i. e. the bijective case) was first stated by E. Wigner in 1931. In his book Wigner himself did not give a rigorous mathematical proof, he said "it is trivial". However, the first such proof (for the bijective case) was given by J. S. Lomont and P. Mendelson, thirty-two years later. One year after that V. Bargmann gave another proof. Several other proofs were given so far to the bijective and non-bijective versions.

In my talk I would like to present a completely new, elementary and very short proof of this famous theorem which is very important in quantum mechanics. We do not assume bijectivity of the mapping or separability of the underlying space like in many other proofs. The advantages of our approach is that it is short, there are no hard calculations, it is very elementary and it works for the general case. As far as we know, such a proof which shares all of these properties was never given before.

Complete decomposability of positive compact operators with positive commutators

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As an application of the Lomonosov theorem one can easily see that every commutative family of compact operators on a Banach space is triangularizable. In the case of Banach lattices the order analog does not exist since there exist indecomposable positive compact operators. In this talk we consider positive commutators of positive compact operators. It is known that such commutator is necessarily quasinilpotent. This immediately implies that a semigroup $\mathcal{S}$ of positive compact operators with the property that every commutator between operators from $\mathcal{S}$ is positive or negative is triangularizable. The natural question that arises is complete decomposability of such semigroups. We will present positive results for general families, semigroups and Lie sets of compact operators.

This is a joint work with R. DRNOVŠEK (University of Ljubljana).

Local and global liftings of analytic families of idempotents in Banach algebras

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Generalizing results of our earlier paper, we investigate the following question. Let $\pi(\lambda) : A \to B$ be an analytic family of surjective homomorphisms between two Banach algebras, and $q(\lambda)$ an analytic family of idempotents in B. We want to find an analytic family $p(\lambda)$ of idempotents in A, lifting $q(\lambda)$, i.e., such that $\pi(\lambda)p(\lambda) = q(\lambda)$, under hypotheses of the type that the elements of $\operatorname{Ker} \pi(\lambda)$ have small spectra. For spectra which do not disconnect $\mathbb{C}$ we obtain a local lifting theorem. For real analytic families of surjective *-homomorphisms (for continuous involutions) and self-adjoint idempotents we obtain a local lifting theorem, for totally disconnected spectra. We obtain a global lifting theorem if the spectra of the elements in $\operatorname{Ker} \pi(\lambda)$ are $\{0\}$, both in the analytic case, and, for *-algebras (with continuous involutions) and self-adjoint idempotents, in the real analytic case. Here even an at most countably infinite set of mutually orthogonal analytic families of idempotents can be lifted to mutually orthogonal analytic families of idempotents. In the proofs, spectral theory is combined with complex analysis and general topology, and even a connection with potential theory is mentioned.

This is a joint work with B. AUPETIT (Université Laval), M. MBEKHTA (Université des Sciences et Technologies de Lille) and J. ZEMÁNEK (Polish Academy of Sciences).

On the length of matrix algebras

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By the *length of a finite system of generators* for a finite-dimensional associative algebra over an arbitrary field we mean the least non-negative integer k such that the words in these generators of lengths not exceeding k span this algebra (as a vector space). The maximum length for the systems of generators of an algebra is referred to as the *length of the algebra*.

Following Paz (1984) we consider the question when the length of a matrix subalgebra can be bounded by a linear function in the order of matrices. We provide linear upper bounds for the lengths of upper-triangular and triangularizable matrix subalgebras over arbitrary fields. We will also present our results on the lengths of commutative algebras.

On median eigenvalues of graphs

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Median eigenvalues of a graph are closely related to the HOMO-LUMO separation properties from mathematical chemistry. The talk will give an overview of recent results about the median eigenvalues. One particular surprising discovery is that the median eigenvalues of every connected bipartite graph G of maximum degree at most three belong to the interval $[-1, 1]$ with a single exception of the Heawood graph, whose median eigenvalues are $\pm\sqrt{2}$. Moreover, if G is not isomorphic to the Heawood graph, then a positive fraction of its median eigenvalues lie in the interval $[-1, 1]$. This result has a generalization to larger vertex degrees, where the only exceptions appear to be point-line incidence graphs of finite projective planes.

The total graphs of finite commutative semirings

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Recently, a lot of study of algebraic structures has been explored via the graph theoretic approach. In the talk, we introduce the total graph of a semiring as the graph with all elements of the semiring as vertices, and two distinct vertices x and y are adjacent if and only if $x + y$ is a zero-divisor.

We present the characterization of finite commutative semirings having the total graph without any 3-cycles. In case the semiring has a cyclic total graph without any 3-cycles, the semiring actually has to be a ring $\mathbb{Z}_2 \times \mathbb{Z}_2$. We also give a characterization of semirings having an acyclic total graph.

This is a joint work with D. DOLŽAN (University of Ljubljana).

The Erdos density theorem revisited

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J.A. Erdos showed that the set of finite rank operators in a nest algebra is dense in the algebra, if an appropriate topology is considered. In this talk, having this density theorem in the background, we investigate the Jordan and the Lie ideal structure of nest algebras, obtaining a complete characterization in the first case and a decomposition theorem in the Lie algebraic setting.

This is partly a joint work with J. ALMEIDA (Instituto Superior Técnico)

On the spectrum and the spectral mapping theorem in max algebra

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We give a new description of spectrum in max algebra of a given non-negative matrix A via local spectral radii and obtain a new block triangular form of A related to its Frobenius normal form. Related results for the usual spectrum of complex matrices and distinguished spectrum for non-negative matrices are also obtained.

As an application we provide a new proof of the spectral mapping theorem in max algebra and also generalize it to the setting of power series in max algebra.

Given a non-negative bounded infinite matrix A, we show that the Bonsall's cone spectral radius of a map $x \mapsto A \otimes x$, with respect to the cone l^{∞}_{+}, is included in its max algebra approximate point spectrum. Moreover, the spectral mapping theorem with respect to point and approximate point spectrum in max algebra is investigated. The corresponding results for more general max and max-plus type kernel operators and for tropical Bellman operators are obtained.

This is a joint work with V. MÜLLER (Czech Academy of Sciences).

Skew-hermitian skew-commuting matrices and fast-decodability of space-time codes

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The existence of $\mathbb{R}$-linearly independent invertible matrices A_i, $i \in I$ satisfying $A_i A_j^* + A_j A_i^*$ for certain distinct pairs $(i, j) \in I \times I$ arises naturally in the context of fast-decodability of space-time codes, which are a family of codes used in multiple-antenna wireless transmission. We describe the context, and then describe some results about the existence of such pairs. We show that the best-case decoding complexity of a full rate space-time code is unfortunately quite high, of the order of $|S|^{n^2+1}$ where S is the signal set and n is the number of antennas. Interestingly, we use the theory of Azumaya algebras to give a mild generalization of the Eckmann-Hurwitz-Radon bounds on the existence of pairwise skew-commuting matrices.

This is a joint work with G. BERHUY (Université Joseph Fourier) and N. MARKIN (Nanyang Technological University).

Patterns of symmetric matrices that allow all the eigenvalue multiplicities to be even

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Let G be an undirected graph on n vertices and let $S(G)$ be the set of all real symmetric $n \times n$ matrices whose nonzero off-diagonal entries occur in exactly the positions corresponding to the edges of G. The Inverse Eigenvalue Problem for a graph G is a problem of determining all possible lists that can occur as the lists of eigenvalues of matrices in $S(G)$. This question is, in general, hard to answer and several variations of the problem have been studied, most notably the minimum rank problem. In this talk, we will discuss some questions related to the Inverse Eigenvalue Problem for a graph related to the possible multiplicities of the eigenvalues that can occur for a matrix in $S(G)$. Of particular interest will be the question, when there exists a matrix $A \in S(G)$ whose eigenvalues all have even multiplicities.

This is a joint work with P. OBLAK (University of Ljubljana).

Identities of matrix algebras

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We will consider various types of identities of matrix algebras. Trace identities are a generalization of polynomial identities, and every trace identity is a consequence of the Cayley-Hamilton identity by the second fundamental theorem of matrix invariants. We will show that this is no longer the case for quasi-identities, while it holds for functional identities.

This talk will be based on a joint work with M. BREŠAR (University of Ljubljana) and C. PROCESI (University of Rome).

Open linear maps and geometry of the numerical range

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The maximum-entropy principle goes back to Boltzmann's pioneering ideas in statistical mechanics and von Neumann has generalized the maximum-entropy principle to quantum systems where it plays a major role in quantum statistical mechanics and quantum state reconstruction. Given the expected values of a set of observables, the *maximum-entropy inference* solves the inverse problem of selecting a quantum state which satisfies the expected value constraints by maximizing an entropic functional.

A. Knauf and myself have observed that the maximum-entropy inference can be discontinuous [1]. This can never happen for commutative observables. I show in [2] that the continuity of a family of constrained optimization problems is equivalent to the *openness* of the projection to the constraint sets. For a family of parallel linear constraints on a convex body I provide sufficient conditions for the openness. They solve for certain size-three block-diagonal matrices the continuity problem of the maximum-entropy inference. A discussion for arbitrary size-three matrices is still in its initial stages.

The set of expected value pairs of two observables equals the *numerical range* [3]. From the perspective of the Grassmannian of two-dimensional planes, spanned by pairs of observables, a discontinuous inference appears where non-exposed points of the numerical range disappear [1]. This holds for certain size-three block-diagonal matrices and I would like to discuss this problem more broadly by invoking the continuous—open equivalence and geometric results about the numerical range [3].

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Contributed talks at IWMS

Simulating data to demonstrate that the Integrated Likelihood Method (ILM) works for parameter estimation when some data values are missing at random

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To demonstrate the efficacy of the method data are simulated from a multivariate normal distribution with known parameters. Then a proportion of the values are deleted at random. The resulting data are analyzed using ILM. This demonstrates that the parameter estimate obtained are consistent with the true values. This can be verified by increasing the size of the data sets and by using different parameter values.

On the BLUEs in two linear models via C. R. Rao's Pandora's box

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Partitioned linear models are used in the estimations of subparameters in regression models as well as in the investigations of some submodels and reduced models associated with the original model. In this study, we consider the estimation of the parameters in two partitioned linear models, denoted by $\mathcal{A} = \{\mathbf{y}, \mathbf{X}_1\boldsymbol{\beta}_1 + \mathbf{X}_2\boldsymbol{\beta}_2, \mathbf{V}_{\mathcal{A}}\}$ and $\mathcal{B} = \{\mathbf{y}, \mathbf{X}_1\boldsymbol{\beta}_1 + \mathbf{X}_2\boldsymbol{\beta}_2, \mathbf{V}_{\mathcal{B}}\}$, which we call full models. Correspondingly, we define submodels $\mathcal{A}_1 = \{\mathbf{y}, \mathbf{X}_1\boldsymbol{\beta}_1, \mathbf{V}_{\mathcal{A}}\}$ and $\mathcal{B}_1 = \{\mathbf{y}, \mathbf{X}_1\boldsymbol{\beta}_1, \mathbf{V}_{\mathcal{B}}\}$. Using the so-called Pandora's Box approach introduced by Rao [C. R. Rao, Unified theory of linear estimation, Sankhyā Ser. A 33, 371–394 (1971)], we give new necessary and sufficient conditions for the equality between the best linear unbiased estimators (BLUEs) of $\mathbf{X}_1\boldsymbol{\beta}_1$ under $\mathcal{A}_1$ and $\mathcal{B}_1$ as well as under $\mathcal{A}$ and $\mathcal{B}$. In our considerations we will utilise the Frisch–Waugh–Lovell theorem which provides a connection between the full model $\mathcal{A}$ and the reduced model $\mathcal{A}_r = \{\mathbf{M}_2\mathbf{y}, \mathbf{M}_2\mathbf{X}_1\boldsymbol{\beta}_1, \mathbf{M}_2\mathbf{V}_{\mathcal{A}}\mathbf{M}_2\}$ with $\mathbf{M}_2$ being an appropriate orthogonal projector. Moreover, we consider the equality of the BLUEs under the full models assuming that they are equal under the submodels. We note that considering the problems of linear estimation from linear statistical models by means of the Pandora's Box approach have some advantages from the computational point of view since estimation and inference from a linear model reduces to the computation of a generalized inverse of the matrix given in the Pandora's Box equation as also noted by Rao (1971).

This is a joint work with S. PUNTANEN (University of Tampere) and H. ÖZDEMİR (University of Sakarya).

The computation of the group inverse and related properties of Markov chains via perturbations

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The derivation of the group inverse and the mean first passage times in a finite ergodic Markov chain is explored. The basic technique involves row by row perturbations of the transition matrix with a systematic update at the each stage. By starting from a simple base where no formal computations are required, six different algorithms are compared for accuracy. The techniques are based on those outlined in Hunter, J. J., The computation of stationary distributions of Markov chains through perturbations, Journal of Applied Mathematics and Stochastic Analysis, 4, 29-46, (1991).

An application of the generalised JLS model on different stock market indices and the 2007–2008 financial crisis

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Sornette and co-workers proposed that, prior to crashes, the stock index price time series is characterised by the Log-Periodic Power Law (LPPL) equation of the Johansen–Ledoit–Sornette (JLS) model. In this paper, we use a Differential Evolution algorithm for global optimisation of the highly nonlinear JLS model. We analyse the JLS model's residuals and propose an ARMA/GARCH error model to capture the residuals' behaviour. Furthermore, we use the extended autocorrelation function (EACF) method for an order determination of the ARMA/GARCH model and compare these results with those of the Akaike and Bayesian Information Criteria. The original JLS model and its generalisation are applied to the well-documented crash of October 1987 of the indices S&P 500 and Dow Jones Industrial Average, and to the DAX index prior to the crash of 1998. We also provide empirical results to show that

these models could have been used to predict the 2007–2008 financial crisis. Moreover, we show that our generalised JLS model improves the statistical properties of the model residuals.

This is a joint work with M. OMLADIČ (University of Ljubljana).

Using the Gram-Schmidt construction to develop linear models

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This talk describes how the Gram-Schmidt construction can be used to simplify and unify the development of the basic linear algebra results required for statistical inference in the Gauss-Markov model.

Studying the singularity of LCM-type matrices via semilattice structures and their Möbius functions

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The invertibility of LCM matrices and their Hadamard powers have been studied a lot over the years by many authors. Bourque and Ligh conjectured in 1992 that the LCM matrix $[S] = [[x_i, x_j]]$ on any GCD closed set $S = \{x_1, x_2, \ldots, x_n\}$ is invertible, but in 1997 this was proven false by Haukkanen et al. However, currently there are many open conjectures concerning LCM matrices and their real Hadamard powers presented by Hong. In this presentation we utilize lattice-theoretic structures and the Möbius function to explain the singularity of classical LCM matrices and their Hadamard powers. At the same time we end up disproving some of Hong's conjectures. We apply the mathematics software Sage to show that every 8-element GCD closed set S, for which the LCM matrix [S] is singular, has the same semilattice structure. We also construct a GCD closed set S of odd numbers such that the LCM matrix [S] is singular. Elementary mathematical analysis is applied to prove that for most semilattice structures there exist a set $S = \{x_1, x_2, \ldots, x_n\}$ of positive integers and a real number $\alpha > 0$ such that S possesses this structure and the power LCM matrix $[[x_i, x_j]^\alpha]$ is singular.

This is a joint work with P. HAUKKANEN (University of Tampere) and J. MÄNTYSALO (University of Tampere).

Restricted kernel canonical correlation analysis

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Kernel canonical correlation analysis (KCCA) is a procedure for assessing the relationship between two sets of random variables when the classical method, canonical correlation analysis (CCA), fails because of the nonlinearity of the data. The KCCA method is mostly used in machine learning, especially for information retrieval and text mining. Because the data is often represented with non-negative numbers, we propose to incorporate the non-negativity restriction directly into the KCCA method. Similar restrictions have been studied in relation to the classical CCA and called restricted canonical correlation analysis (RCCA), so that we call the proposed method *restricted kernel canonical correlation analysis* (RKCCA).

With the Karush-Kuhn-Tucker theorem we show that the solution of RKCCA equals an unconstrained solution to a modified CCA problem on two random vectors with known covariance matrix where one or several variables have been excluded. Furthermore we use the idea of sub-vectors and sub-matrices to translate the problem of searching for the kernel canonical correlation under non-negativity restriction into an optimization problem related to eigenvalues of some generalized eigenvalue problem with a real symmetric matrix and a positive definite matrix.

Random walks relative to multiple transition matrices

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In discrete time $0, 1, \dots, k$ a particle is travelling between a finite set of states relative to a transition matrix and constitutes a random walk of length k. Given the cost matrix corresponding to transitions between states, the mean of the cost along a random walk of length k starting at some specified state needs to be computed in many applications. In this talk, we first introduce a generalization of the above model for multiple transition and cost matrices, and then propose Monte Carlo techniques to get approximation of the mean by using random numbers and simulation. Experiments on artificial data are conducted to evaluate the performance of the presented approaches in comparison with the one that uses diffusion wavelets method for computing powers of matrices.

Links between fixed linear model and mixed linear model

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The linear mixed model has strong links with a particular augmented linear model including only fixed effects. This goes back to Henderson's mixed model equations. In this talk we point out that the connection between the two models is actually very straightforward: a mixed linear model can be obtained from the augmented model by a simple linear transformation. This immediately opens up a new viewpoint for studying the relationship between the BLUEs and BLUPs in the two models.

This is a joint work with B. ARENDACKÁ (Physikalisch-Technische Bundesanstalt).

The minimax copulas

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The theory of copulas started with Sklar's theorem proposing a universal model for expressing dependence of random variables. With the range of applications in applied mathematics expanding and varying from mathematics of finance to system theory, there is a growing need for new types of copulas that could serve as appropriate models in these applications. It is our aim to set a counterpart to the famous Marshall copulas (an extension of Marshall-Olkin copulas) that are typically applied to model lifetime of a two-component system where components are subject to "shocks". Even a small but essential change in the problem that the model is applied to such a system with one of the components having a backup option leads to possibly quite different copulas. So, it is our goal to construct copulas that model dependence of random variables $U = \max\{X, Z\}$ and $V = \min\{Y, Z\}$ where X, Y and Z are independent random variables. We will present a full study of the augmented case by introducing a new family of copulas, called minimax copulas, together with some of their properties and examples.

This is a joint work with M. OMLADIČ (University of Ljubljana).

On a new family of weighted total least-squares algorithms for EIV-models with arbitrary dispersion matrices

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For a long time, algorithms for the Total Least-Squares (TLS) solution within Errors-In-Variables (EIV) Models would only tolerate "element-wise weighting," which essentially amounts to the use of diagonal dispersion matrices without auto- or cross-correlations. This dilemma was overcome first by Schaffrin and Wieser (2008), and later by Fang (2011) as well as Mahboub (2012), who all allowed to handle more general dispersion matrices, while assuming invertibility and/or lack of cross-covariances.

Finally, in his PhD dissertation, K. Snow (2012) designed an algorithm that would generate the TLS solution even if the dispersion matrices are singular and cross-covariances exist, as long as a certain uniqueness criterion is fulfilled. Here, a new but related family of algorithms will be presented that are able to generate the (properly weighted) TLS solution with greater efficiency.

This is a joint work with K. SNOW (The Ohio State University and Topcon Positioning Systems).

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Inheritance properties of generalized Schur complements and principal pivot transforms of matrices

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The notions of Schur complement and principal pivot transforms have been studied quite extensively in matrix theory. The importance of the Schur complement in problems arising in areas including numerical analysis and statistics is well documented in the literature. The notion of the principal pivot transform appears to have its roots in the theory of the linear complementarity problem. Many results on the preservation of matrix classes like the P-property are well known. Studies also have been extended to the case of the Moore-Penrose inverse in the definitions of the Schur complement and the principal pivot transform. In this talk, we report new results on the inheritance properties of these generalizations, in the context of certain matrix classes.

This is a joint work with K. BISHT (IIT Madras) and G. RAVINDRAN (ISI Chennai).

Enhancing Gibbs sampling method for motif finding in DNA with initial graph representation of sequences

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Finding short patterns with residue variation in a set of sequences is still an open problem in genetics, since motif finding techniques on DNA and protein sequences are inconclusive on real data sets and their performance varies on different species. Hence finding new algorithms and evolving established methods are vital to further understand genome properties and the mechanisms of protein development. In this talk we present an approach to search for possible motifs in connection to Gibbs sampling method. Starting points in the search space are partly determined via graphical representation of input sequences opposed to completely random initial points with the standard Gibbs sampling. Our algorithm is evaluated on synthetic as well as on real data sets by using several statistics, such as sensitivity, positive predictive value, specificity, performance and correlation coefficient. Additionally, a comparison between our algorithm and basic standard Gibbs sampling algorithm is made to show improvement in accuracy, repeatability and performance.

Estimation of the covariance matrix based on two types of the forward search algorithm

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Multivariate normal distribution is a crucial assumption in many statistical models and estimation of the population covariance matrix is usually the first key step in modelling multivariate data. The standard sample covariance matrix is commonly used but it gives unreliable estimates if the normal distribution does not fit the data.

Forward search algorithm is an iterative and graphical method for data exploration and robust parameter estimation [1]. The algorithm orders the data according to their distances form the underlying distribution or model. The nearest observations form the initial basic set, which is very robust. The basic set is then increased in size step by step until all observations are included. Parameter estimates and different statistics are computed with the basic sets of increasing sizes.

We will demonstrate the use of the forward search algorithm in the context of robust covariance matrix and confirmatory factor model estimation [2,3]. The former uses Cook's distance to measure the influence of an observation and the later uses observational residuals. The comparison of the two methods applied to real and simulated data sets will show that outliers and influential observations can be model specific. Model-based robust techniques should therefore be emphasized.

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Solving a 6 x 6 Survo puzzle using matrix combinatorial products

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We apply a new computational method for solving a demanding 6×6 Survo puzzle with binary matrices that are recoded and combined using the Hadamard, Kronecker, and Khatri–Rao products. An extra challenge is provided by readily given numbers that make the puzzle solvable.

This is a joint work with R. SUND (University of Helsinki).

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On R^2 in linear mixed models

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The statistic R^2 in fixed-effects regression settings is mostly understood as a measure of the proportion of variability explained by the model. Here we look into several definitions of generalizations of R^2 defined for linear mixed effects models published during recent decades. We try to address questions such as "Do these measures coincide in specific models?" "What do they measure?", etc.

This is joint work with O. BLAHA (LSUHSC School of Public Health) and L. R. LAMOTTE (LSUHSC School of Public Health).

Some results on permutations of matrix products

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It is well-known that trace$(AB) \geq 0$ for real-symmetric nonnegative definite matrices A and B. However, trace(ABC) can be positive, zero or negative, even when C is real-symmetric nonnegative definite. The genesis of the present investigation is consideration of a product $A = A_1 A_2 \cdots A_n$ of square matrices. Permuting the factors of A leads to a different matrix product. We are interested in conditions under which the spectrum remains invariant. The main results are for square matrices over an arbitrary algebraically closed commutative field. The special case of real-symmetric, possibly nonnegative definite, matrices is also considered.

This is a joint work with I. OLKIN (Stanford University).

Working groups

Spectral variation and geometry of the normal matrices

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Given normal matrices A and B with eigenvalues a_k and b_k we define the spectral distance $\mathrm{sd}(A, B)$ as $\min(\max |a_k - c_k|)$ where c_k runs over all permutations of b_k. The spectral variation inequality $\mathrm{sd}(A, B) \le \|A - B\|$ (operator norm) holds in several important special cases, but fails in general. In its place we have $\mathrm{sd}(A, B) \le K\|A - B\|$ where K is a universal constant that is approximately 2.9. However, 2.9 is much larger than any known examples would suggest. A possible way to improve such variation bounds lies in the observation (first made long ago by Rajendra Bhatia) that $\mathrm{sd}(A, B)$ is less than the arc-length of any normal path from A to B. This directs our attention to the problem of finding geodesics (with respect to the operator norm) in the space of normal matrices (and this is also a natural problem independent of applications). Remarkably, to our knowledge no normal geodesics have been identified for sure aside from "short normal paths", ie normal paths from A to B that have the minimum possible length $\|A - B\|$ (although they are not typically straight-line paths). Short normal paths exist, for example, when the eigenvalues of A and B lie on concentric circles (this includes all 2×2 cases). Otherwise the normal geodesic problem appears to be wide open! Even some progress on the 3×3 case would be welcome.

Special invariant subspaces, local-to-global properties and approximation problems

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One or more questions of the following type will be considered.

1. LOCAL-TO-GLOBAL PROPERTIES

If every member of a collection of operators (e.g., a group, semigroup, algebra) has a certain property, then under what conditions does the collection have it simultaneously? A. Simonic (1992) gave an answer to the question: If every member of a group G of matrices is (individually) similar to a positive definite matrix, is G simultaneously similar to a group of positive definite matrices? His answer is no in general, but yes if G is divisible. One question to ponder: change "positive definite" to "positive" or "nonnegative" in the question above. "Nonnegative" means that every entry of the matrix is nonnegative. More generally, an operator on a function space is said to be nonnegative if it takes the set of nonnegative functions into itself. Another question of this type: If each member of a semigroup S of matrices has the increasing spectrum property (see below for definition), is S simultaneously triangularizable by a permutation similarity?

2. Standard Invariant Subspaces

Let L be a space of functions on X (e.g., an L_p space with a given measure on the underlying space X). If Y is a subspace of X, and M the set of all functions supported on Y, then we call M a standard subspace of L. If an operator (or a collection of operators simultaneously) has a nontrivial standard invariant subspace, it is called decomposable. If the standard invariant subspaces for a collection C form a maximal subspace chain, then C is said to have a standard triangularization. We say that an operator T on L has the increasing spectrum property if whenever M and N are standard subspaces of L with $M < N$, then the spectrum of the compression of T to M is contained in the spectrum of its compression to N. (In finite dimensions this means, for a given matrix, that if you take two principal submatrices one contained in the other, then the set of eigenvalues of the smaller submatrix is contained in that of the larger one–which is obviously the case if the matrix is triangular.) Question: If K is a compact operator on an L_2 space and if K has the increasing spectrum property, does K have a standard triangularization? In particular, is K decomposable? L. Marcoux, M. Mastnak, and H. Radjavi (2009) asked the question and showed that the answer is yes if K has finite rank.

3. From Positive to General

Let M be a nonnegative matrix (in the sense of paragraph 1). If the diagonal of M consists exactly of its eigenvalues with the right multiplicities, then M is triangular after a similarity by a permutation. This was extended to infinite-dimensional setting by J. Bernik, L. Marcoux, and H. Radjavi (2012). What about general operators–not necessarily nonnegative? The short answer is easily: no, except in dimension 2. But we are looking for long answers!

4. Almost nilpotent vs nearly nilpotent

An operator $T \in \mathbb{M}_n(\mathbb{C})$ is said to be **almost nilpotent of order** $1 \leq k \leq n$ if $\|T^k\|$ is very small. Let $\mathcal{N}_n^{(k)} = \{N \in \mathbb{M}_n(\mathbb{C}) : N^k = 0\}$. We say that T is **nearly nilpotent of order** k if $\mathrm{dist}(T, \mathcal{N}_n^{(k)})$ is small. Can one show that there exists a function $f : (0, \infty) \to (0, \infty)$, *independent of n*, such that

- $\lim_{x\to 0^+} f(x) = 0$ and
- $\|T^k\|^{1/k} < \varepsilon$ implies $\mathrm{dist}(T, \mathcal{N}_n^{(k)}) < f(\varepsilon)$?

That is, can one show that every almost nilpotent operator of order k is nearly nilpotent of order k? It would still be very, very interesting if one could show that there exists a second function $\mu : \mathbb{N} \to \mathbb{N}$ *independent of* n so that

$$\|T^k\|^{1/k} < \varepsilon \quad \text{implies that} \quad \mathrm{dist}(T, \mathcal{N}_n^{(\mu(k))}) < f(\varepsilon).$$

A positive answer to this question would resolve a question from the PhD Thesis of Lawrence Williams (circa 1976): if $Q \in \mathcal{B}(\ell_2)$ is quasidiagonal and quasinilpotent, is Q a limit of block-diagonal nilpotents? (Definitions available upon request.)

5. EXTREMELY NON-NORMAL OPERATORS

We shall say that an operator $T \in \mathbb{M}_n(\mathbb{C})$ is **extremely non-normal** if $[T, T^*] := TT^* - T^*T$ is invertible (and thus, in some sense, as "far away" from normal operators as possible). For example, when $n = 2$, then every operator is either normal or extremely non-normal. (Upper-triangularize T and calculate the determinant of $[T^*, T]$.) This fails when $n = 3$, since $[1] \oplus E_{12}$ is neither normal nor extremely non-normal, where $E_{12} \in \mathbb{M}_2(\mathbb{C})$ denotes the usual $(1, 2)$-matrix unit. Can one characterize extremely non-normal operators in $\mathbb{M}_n(\mathbb{C})$ for $n \geq 3$?

Topics in quaternion linear algebra

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A working group is proposed with focus on various topics of matrix analysis for matrices with entries in the skew field of real quaternions. Among possible topics: numerical ranges, connections between classical equivalence relations with respect to real, complex, and quaternion matrices, and applications.