Adjacency preservers on symmetric matrices over a finite field

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Let $S_n(\mathbb{F}_q)$ denote the set of all symmetric $n \times n$ matrices over a finite field $\mathbb{F}_q$ with q elements. A map $\Phi : S_n(\mathbb{F}_q) \to S_n(\mathbb{F}_q)$ *preserves adjacency in both directions* if

$$\text{rank}(A - B) = 1 \Longleftrightarrow \text{rank}(\Phi(A) - \Phi(B)) = 1 \tag{1}$$

for all $A, B \in S_n(\mathbb{F}_q)$. Bijective maps that preserve adjacency in both directions are classified by the fundamental theorem of geometry of symmetric matrices. In this talk we present a result, which states that any map Φ that preserves adjacency in one direction (i.e. the equivalence in (1) is replaced by an implication) is necessary bijective if $n \geq 3$. This is not true if $n = 2$, regardless of the value q. The proof relies on some results from graph theory and finite field theory.