Discrete-time stability of polynomial matrices

Harald Wimmer, University of Würzburg, Germany

Polynomial matrices $G(z) = Iz^m - \sum_{i=0}^{m-1} C_i z^i$ with normal or hermitian coefficients C_i are studied.

Results on block diagonal stability and discrete–time Lyapunov equations are used to extend the following theorem from polynomials to polynomial matrices.

Let $g(z) = z^m - \sum_{i=0}^{m-1} c_i z^i$ be a real polynomial. Suppose $c_0 \neq 0$ and

$$\sum_{i=0}^{m-1} |c_i| \leq 1.$$

Then $\rho(g) = \max\{|\lambda|;\, g(\lambda) = 0\} \leq 1$. If λ is a root of $g(z)$ with $|\lambda| = 1$ then λ is a simple root and $\lambda^d = \pm 1$ for some d with $d \,|\, m$. If $\rho(g) = 1$ then either $g(1) = 1$ and

$$g(z) = (z^k - 1) f(z^k)$$

or $g(1) \neq 1$ and

$$g(z) = (z^k + 1) f(z^k),$$

and $f(\mu) \neq 0$ if $|\mu| = 1$.

Two applications are discussed, namely an Eneström–Kakeya theorem for polynomial matrices and a stability and convergence result for a system of difference equations.