

Non axiomatizability of real spectra and algebraic characterization of completely normal Zariski spectra

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(Joint work with T. Mellor and N. Schwartz)

Delzell and Madden have constructed a completely normal spectral space that is not the real spectrum of any commutative ring. The property of being completely normal has a natural first order axiomatisation in the Stone dual of the space. I will indicate that it is indeed impossible to characterize spectral subspaces of real spectra in a natural first order way.

Furthermore I will introduce an algebraic invariant of a commutative ring A , which expresses complete normality of the Zariski spectrum of A . Since this invariant is not preserved in ultraproducts (of reduced rings), the property of having completely normal Zariski spectra can not be expressed in the first order theory of (reduced) rings.