Eigenvalues of sums of selfadjoint matrices

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(Joint work with H. Bercovici ad W.S. Li)

Suppose we are given three selfadjoint matrices A, B, C, such that $A + B = C$. An old question concerns the determination of the set of possible eigenvalues of C, if the eigenvalues of A and B are given. Horn has conjectured in 1962 that they are characterized by a certain set of inequalities; this very deep conjecture has been proved true in the 90's by work of Klyachko (essentially), Totaro, and Knutson-Tao.

On the other hand, several results had been obtained concerning the location of a single eigenvalue of C. In his survey of the Horn conjecture, published in BAMS in 2000, Fulton has asked the question of the posssible location of a subset of the eigenvalues of C. We give a more general result that completely answers this question, by means of a family of inequalities related to Horn's. As a consequence, a result of Buch (2006) giving conditions for the eigenvalues of Hermitian matrices with positive sum of finite rank is also recaptured.