Low rank perturbations of higher rank numerical ranges

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For a positive integer k, the rank-k numerical range $\Lambda_k(A)$ of an operator A acting on a Hilbert space $\mathcal{H}$ of dimension at least k is the set of scalars λ such that $PAP = \lambda P$ for some rank k orthogonal projection P.

In this talk, the connection between $\Lambda_k(A)$ and $\Lambda_{k-r}(A+F)$, the rank-$(k-r)$ numerical range of A with a perturbation of a rank r operator F, will be discussed. In particular, it can be shown that if A is normal or if the dimension of A is finite, then $\Lambda_k(A)$ can be obtained as the intersection of $\Lambda_{k-r}(A+F)$ for a collection of rank r operators F.

Furthermore, results for the rank-∞ numerical range $\Lambda_\infty(A)$ will also be studied, where $\Lambda_\infty(A)$ is defined as the set of scalars λ such that $PAP = \lambda P$ for an infinite rank orthogonal projection P.