

On varieties of commuting triples

Klemen Šivic, University of Ljubljana, Slovenia

The set $C(3, n)$ of all triples of commuting $n \times n$ matrices over an algebraically closed field F is a variety in F^{3n^2} defined by $3n^2$ equations, which are relations of commutativity. The problem first proposed by Gerstenhaber asks to determine for which natural numbers n this variety is irreducible. This is equivalent to the problem whether $C(3, n)$ equals to the Zariski closure of the subset of all triples of generic matrices (i.e. matrices having n distinct eigenvalues). The answer is known to be positive for $n \leq 7$ and negative for $n \geq 30$. Using simultaneous commutative perturbations of pairs of matrices in the centralizer of the third matrix we prove that $C(3, 8)$ is also irreducible.