

Towards noncommutative real algebraic geometry

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Artin's solution of Hilbert's famous 17th problem on the representation of positive polynomials as sums of squares of rational functions can be considered as the beginning of real algebraic geometry. In present days Positivstellensätze on semi-algebraic sets form a central topic of this field. In the talk we shall propose and discuss how basic concepts (positive elements, semi-algebraic sets, quadratic modules, Archimedean orderings) and results (Positivstellensätze) from real algebraic geometry can be generalized to noncommutative $*$ -algebras. Various Positivstellensätze of $*$ -algebras (Weyl algebras, enveloping algebras, PI-algebras) will be presented.