Archimedean quadratic modules of real polynomials

Alexander Prestel, University of Konstanz, Germany

The polynomials $h_1, \ldots h_s \in \mathbb{R}[X_1, \ldots X_n]$ define an archimedean quadratic module $M(h)$ iff the semialgebraic set $W(h) \subseteq \mathbb{R}^n$ defined by $h_i(x) \geq 0$ for $1 \leq i \leq s$ is bounded and every polynomial $f \in \mathbb{R}[X_1, \ldots, X_N]$, strictly positive on $W(h)$, admits a representation

$$f = \sigma_0 + h_1\sigma_2 + \cdots + h_s\sigma_s$$

with σ_i being sums of squares of polynomials. If all h_i are linear and $W(h)$ is bounded, them $M(h)$ is archimedean. But not all bounded $W(h)$ have archimedean $M(h)$.

There exists an abstract valuation theoretic criterion for $M(h)$ to be archimedean. We are, however, concerned with an effective procedures that allows to decide whether $M(h)$ is archimedean or not. In dimension 2 a geometric such procedure was given by E. Cabral in her PhD thesis. In dimension $n \geq 3$ decidability has now been proved by S. Wagner, but a geometric procedure is still lacking.