Pólya's theorem with zeros

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Let $\mathbb{R}[X] := \mathbb{R}[X_1, \ldots, X_n]$. Pólya's Theorem says that if a form (homogeneous polynomial) $p \in \mathbb{R}[X]$ is positive on the standard n-simplex Δ_n, then for sufficiently large N all the coefficients of $(X_1 + \cdots + X_n)^N p$ are positive. In 2001, Powers and Reznick gave a bound on the N needed, in terms of the degree of p, the coefficients, and minimum of p on Δ_n. This quantitative Pólya's Theorem has many applications, in both pure and applied mathematics. The work is this talk is part of an ongoing project to understand when Pólya's Theorem holds for forms if the condition "positive on Δ_n" is relaxed to "nonnegative on Δ_n", and to give bounds on the N. We prove a "localized" Pólya's Theorem, with a bound on the N needed, which is a quantitative version of a result of Schweighofer. We use this result to give a sufficient condition for forms which are non-negative on Δ_n to satisfy the conclusion of Pólya's Theorem, with a bound on the N needed.