On the irreducibility of commuting varieties

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In my talk, I am going to review the irreducibility problem for various types of "commuting varieties". The basic example is the variety $C = C(2, n)$ of pairs of commuting n by n complex matrices. It is well-known that $C(2, n)$ is irreducible. There are several directions for generalising this situation. One can consider

1) arbitrary reductive Lie algebras in place of the matrix algebra $gl(n)$;

2) commuting varieties associated with non-reductive subalgebras of $gl(n)$;

3) triples, quadruples, etc. of matrices, i.e., varieties $C(3, n)$, etc.

4) intersection of C with subspaces of $gl(n) \times gl(n)$;

5) subvarieties of C determined by natural polynomial conditions.

In many cases, the varieties obtained appear to be reducible, and the natural problem is to describe their irreducible components. We also discuss some open problems related to commuting varieties of matrices.