Faithful linear representations of bands

Jan Okniński, Warsaw University, Poland

(Joint work with Ferran Cedo)

A semigroup S such that $a^2 = a$ for every $a \in S$ is called a band. The main motivating problem for this talk is to find conditions on a band S in order that S embeds into the multiplicative semigroup $M_n(F)$ of $n \times n$ matrices over a field F for some $n \geq 1$. It is known and easy to show that this is always the case if S is a rectangular band (that is, a semigroup satisfying the identity $xyx = x$), but this is no longer true in general (that is, in case the band S has at least 2 rectangular band components). The following related problem will be also discussed: when the semigroup algebra $K[S]$ of a band S over a field K is embeddable into $M_n(A)$ for a commutative algebra A? Certain general results will be proved and some concrete embeddings will be constructed.