Blum-Hanson property and quasisimilarity of operators

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Let T be a contraction on a Hilbert space H such that T^n converges in the weak operator theory. By a result motivated by ergodic theory then T has the Blum-Hanson property, i.e., $\lim_{N \to \infty} N^{-1} \sum_{n=1}^{N} T^{k_n} x$ exists in the norm topology for each $x \in H$ and each increasing subsequence (k_n). We show that this is not true for power bounded Hilbert space operators. This also implies that there are power bounded operators which are not quasisimilar to a contraction.

This answers an open problem from ergodic theory as well as questions concerning (quasi)similarity of operators.