Real solving polynomial equations with semidefinite programming

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While good methods exist for computing complex roots to polynomial equations, the problem of computing all *real* solutions is less well understood. We propose a numerical method for finding the real solutions to a system of polynomial equations $h_1 = 0, \ldots, h_m = 0$, assuming their number is finite (while the number of complex roots could be infinite). Our method relies on expressing the real radical ideal $\sqrt[\mathbb{R}]{I}$ of the ideal I generated by $h_1, \ldots, h_m$ as the kernel of a suitable positive semidefinite moment matrix. We use semidefinite optimization for finding such a matrix, combined with linear algebra techniques for computing the real roots as well as a (border or Gröbner) basis of the ideal $\sqrt[\mathbb{R}]{I}$.

The same method can be adapted to find the complex roots of I, simply by omitting the positive semidefinite condition, in which case it returns a border base of an ideal J nested between I and its radical $\sqrt{I}$, with $J = I$ precisely when $\mathbb{R}[x]/I$ is a Gorenstein algebra. Using recent ideas from Janovitz-Freireich et al. we can then also find a base of the radical ideal $\sqrt{I}$.

It turns out that our stopping criterion (based on some flatness assumption for moment matrices) is closely related to the stopping criterion used by Zhi and Reid in their algorithm for complex roots inspired by involutive methods for systems of linear PDE's. Moreover, the Zhi-Reid algorithm can be adapted to finding real roots by adding positive semidefinite constraints.