Some new bounds on realizing spectra in the nonnegative inverse eigenvalue problem

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(Joint work with Helena Šmigoc)

Let $\sigma := (\lambda_1, \quad \dots \quad, \lambda_n)$ be a list of complex numbers. The nonnegative inverse eigenvalue problem (NIEP) asks for necesssary and sufficient conditions on σ to be the spectrum of a matrix with nonnegative real entries. We will summarize the literature on this problem and present some new sufficient results. Suppose that

$$\text{maximum}\{\mid \lambda_j \mid \quad : \quad j = 1, 2, \quad \dots \quad n\}$$

is a non-repeated element of σ. Let

$$s_k := \lambda_1^k + \quad \dots \quad + \lambda_n^k, \quad k = 1, 2, 3, \dots.$$

be the associated Newton power sums. Then a celebrated result of Boyle and Handelman states that if all the s_k are positive, then there exists a nonnegative integer N such that

$$\sigma_N := (\lambda_1, \quad \dots \quad, \lambda_n, 0, \quad \dots \quad, 0), \qquad (N \text{ zeros})$$

is the spectrum of a nonnegative $(n+N) \times (n+N)$ matrix. The problem of obtaining a constructive proof of this result with an effective bound on the minimum number N of zeros required has not yet been solved. We will discuss an approach to this using polynomials, and apply it to the test spectrum $(3+t, 3-t, -2, -2, -2)$. In particular, we will show that, for $t > 0$, if

$$N > 2\log_3(\frac{2}{t}),$$

then

$$(3+t, 3-t, -2, -2, -2, 0, \quad \dots \quad, 0), \qquad (N \text{ zeros})$$

is the spectrum of a nonnegative $(n+N) \times (n+N)$ matrix.