

## Supertropical semirings and supervvaluations

Manfred Knebusch, University of Regensburg, Germany

(Joint work with Louis Rowen and Zur Izhakian)

It has long been felt that along with the now rapidly evolving tropical geometry there should be developed a fitting tropical algebra. I will report on a new approach to tropical algebra, still very much work in progress with Louis Rowen and Zur Izhakian, both at Bar Ilan University, Tel Aviv. In contrast to established tropical algebra with its max-plus semirings, we build and study “supertropical” semirings. Here the sum of two elements, which in the max-plus setting would be zero, is a “ghost” element. Intuitively a ghost element is near to zero, but not exactly zero. Every “tangible” element of the semiring has an associated ghost. Usually one obtains results, e.g identities, which are only true up to ghost elements. This is the prize we have to pay for the fact, that we have no subtraction in semirings. The prize is definitely lower than in usual tropical algebra. The supertropical approach seems to be particularly apt to deal with valuations. To any valuation on a ring  $R$  (in the usual Bourbaki sense) we associate a “supervaluation” on  $R$ , which takes values in a supertropical semiring. This allows us to ask new questions about valuations, already in the case of fields. If time allows, I will present a result on convex valuations on a ring, which can be stated completely within usual real algebra, and probably has escaped the attention of the not supertropically minded real algebraists.