Norm inequalities for commutators of self-adjoint operators

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Let A, B, and X be bounded linear operators on a complex separable Hilbert space. It is shown that if A and B are self-adjoint with $a_1 \leq A \leq a_2$ and $b_1 \leq B \leq b_2$ for some real numbers a_1, a_2, b_1, and b_2, then for every unitarily invariant norm $|||\cdot|||$,

$$|||AX - XB||| \leq \max(a_2 - b_1, b_2 - a_1) \, |||X||| \, .$$

If, in addition, X is positive, then

$$|||AX - XA||| \leq \frac{1}{2}(a_2 - a_1) \, |||X \oplus X||| \, .$$

These norm inequalities generalize recent related inequalities due to Kittaneh, Bhatia-Kittaneh, and Wang-Du.