When Jordan submodules are bimodules

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(Joint work with Matej Brešar and Victor Shulman)

Let $\mathcal{A}$ be an associative algebra and let X be an $\mathcal{A}$-bimodule. We call a linear subspace Y of X a Jordan $\mathcal{A}$-submodule of X if $Ay + yA \in Y$ for all $A \in \mathcal{A}$ and $y \in Y$ (if $X = \mathcal{A}$, then this coincides with the classical concept of a Jordan ideal). When is a Jordan $\mathcal{A}$-submodule a submodule? We give a thorough analysis of this question in both algebraic and analytic context. We consider general algebras and general Banach algebras and treat some more specific topics, such as symmetrically normed Jordan $\mathcal{A}$-submodules. Some of our results are of interest also in the classical situation; in particular, we show that there exist C^*-algebras having Jordan ideals that are not ideals.