The pp conjecture in the theory of spaces of orderings

Pawel Gladki, University of California, Santa Barbara, USA

Spaces of orderings were introduced by Murray Marshall in the 1970's and provide an abstract framework for studying orderings on fields and the reduced theory of quadratic forms over fields. Numerous important notions in this theory, such as isometry, isotropy, or being an element of a value set of a form, make an extensive use of positive primitive formulae in the language of special groups. Therefore, the following question, which can be viewed as a type of very general local-global principle, is of great importance: is it true that if a positive primitive formula holds in every finite subspace of a space of orderings, then it also holds in the whole space? This problem is now known as the pp conjecture. The answer to this question is affirmative in many cases, although it has always seemed unlikely that the conjecture has a positive solution in general. In this talk we shall discuss first counterexamples for which the pp conjecture fails – namely, we shall classify spaces of orderings of function fields of rational conics with respect to the pp conjecture, and show for which of such spaces the conjecture fails, and then discuss the pp conjecture for the space of orderings of the field $\mathbb{R}(x,y)$. The mentioned results are extracted from author's dissertation.