Zero patterns and unitary similarity

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(Joint work with Jinpeng An)

We specify a subspace of $n \times n$ complex matrices by requiring certain entries to vanish. We refer to the collection of the positions of these entries as a zero pattern. We also impose the technical conditions that the zero pattern contains no diagonal entries and that the total number of required zero entries is $n(n-1)/2$. A classic example of such a zero pattern is the lower (or upper) triangular pattern consisting of all entries below (above) the diagonal. Schur's theorem (which is almost 100 years old) asserts that the lower (or upper) zero pattern is universal in the sense that every $n \times n$ complex matrix is unitarily similar to one that admits this zero pattern.

In this talk I shall describe new results proving the existence of many new universal zero patterns. We shall also survey what is known in the cases when n is less than or equal to 4. If time permits, we shall also mention the real and quaternionic versions of our results.

Our paper containing the proofs of the most important results (in a more general setup) is available on arXiv.