

Representation of reduced special groups in algebras of continuous functions

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(Joint work with Francisco Miraglia)

In this talk we address Marshall's long standing *representation problem* for abstract order spaces from a new perspective. Marshall's question is whether any abstract space of orderings is isomorphic (in the category of such spaces) to the space of orderings of a (orderable) Pythagorean field; for background information see [M]. This problem - which admits several meaningful variants - has remained open for over thirty years. There is a prevailing feeling that the answer may well be negative; some even suspect that the statement may be independent of the axioms of set theory. We use here the dual formulation in terms of *reduced special groups* (RSG), cf. [DM].

We show that, upon (slightly?) broadening the class of representation objects from the RSGs of orderable, Pythagorean fields to the RSGs associated with certain rings, *the problem has a positive answer*. More precisely, we show that any RSG is isomorphic to the RSG associated to some multiplicative group of invertible, continuous real-valued functions on a Boolean space X . In particular, the underlying ring, $C(X)$, is a Pythagorean ring with many units which, in addition, is a real closed ring in the sense of Prestel-Schwartz [PS].

References:

[DM] M. Dickmann, F. Miraglia, *Special Groups : Boolean-Theoretic Methods in the Theory of Quadratic Forms*, Memoirs Amer. Math. Soc. 689, Providence, R.I., 2000.

[M] M. Marshall, *Spaces of Orderings and Abstract Real Spectra*, Lecture Notes in Mathematics 1636, Springer-Verlag, Berlin, 1996.

[PS] A. Prestel, N. Schwartz, *Model Theory of Real Closed Rings*, Fields Institute Comm. 32 (2002), 261-290.