Linear Algebra Workshop - Kranjska Gora 08
Abstracts

Zero product preserving maps on $C_1[0,1]$

Jerónimo Alaminos, University of Granada, Spain

We study continuous bilinear maps ϕ from $C_1[0,1] \times C_1[0,1]$ into a Banach space X with the property that $\phi(f,g) = 0$ whenever $fg = 0$. This is applied to the study of zero product preserving operators on $C_1[0,1]$, and operators on $C_1[0,1]$ satisfying a condition similar to locality of an operator.

Remarks on weighted composition operators

Calin Ambrozie, Czech Academy of Sciences, Czech Republic

We consider the question of existence of hyperinvariant subspaces for certain operators of weighted composition with irrational rotations on the space of square integrable functions on the unit circle.

Two parameter eigencurves for definite and indefinite eigenvalue problems

Paul Binding, University of Calgary, Canada

A review will be given of some uses of the embedding $Ax = \lambda Bx - \mu x$, and in particular its (λ, μ) eigencurves, for studying the generalised eigenproblem $Ax = \lambda Bx$. Topics will include some properties of eigencurves; some classes of operators (from classical to recent) that they can accommodate; and some types of spectral questions that they can help to address.

Linear maps preserving the minimum modulus

Abdellatif Bourhim, Laval University, Canada

(Joint work with M. Burgos)

We characterize surjective linear maps that preserve the minimum modulus between unital semisimple Banach algebras, one of them is a unital C^*-algebra having either real rank zero or essential socle.

On a decomposition for pairs of operators

Zbigniew Burdak, Agricultural University of Krakow, Poland

Our aim is to present some result on a decomposition of a pair of operators. More precisely, there are known decompositions of single operators of some classes like decomposition of a contraction (Langer, Nagy, Foias), a isometry (Wold) or a power partial isometry (Halmos, Wallen). For a pair of operators a (joint) decomposition of this type becomes much more complicated (even in case of isometries.) In the presented talk a new decomposition of a pair of commuting, but not necessarily doubly commuting contractions will be proposed. In the case of power partial isometries a more detailed decomposition will be given.

Burdak Z., *On a decomposition for pairs of commuting contractions*, Studia Math. **181** (1) (2007), 33-45.

Sums of hermitian squares as an approach to the BMV conjecture

Sabine Burgdorf, Université de Rennes 1, France

We consider the polynomial $S_{m,k}(X,Y)$ in the noncommuting variables X and Y which is the sum of all monomials of total degree m in which Y appears exactly k times. Besides the trivial cases $k = 0, 1, 2$ we exemplify that for $k = 4$ and arbitrary m the polynomial $S_{m,k}(X^2, Y^2)$ is a sum of hermitian squares and commutators of polynomials in X and Y. Further for $k = 2, 4$ and specific m representations of $S_{m,k}(X,Y)$ as a sum of hermitian squares are given. These results are interesting due to the BMV conjecture which states that the trace of $S_{m,k}(A,B)$ is nonnegative for all positive semidefinite matrices A and B of the same size.

Canonical Hilbert-Burch matrices for ideals of $k[x, y]$

Aldo Conca, University of Genova, Italy

Let k be a field and I be an ideal of $R = k[x, y]$ such that R/I is Artininan, that is, it is of finite dimension as a k-vector space. The Hilbert-Burch theorem implies that I is generated by the t- minors of a matrix A of size $(t+1) \times t$ with entries in R. We call A a Hilbert-Burch matrix for I. An ideal I has many Hilbert-Burch matrices but we show that there is a canonical choice. As an application, we determine the dimension of certain affine Groebner cells related to Hilbert schemes and their Betti strata recovering results of Ellingsrud and Stromme, Goettsche and Iarrobino.

Truncated moment problems: The extremal case

Raúl E. Curto, University of Iowa, USA

For a degree $2n$ real d-dimensional multisequence $\beta \equiv \beta^{(2n)} = \{\beta_i\}_{i \in Z_+^d, |i| \leq 2n}$ to have a *representing measure* μ, it is necessary for the associated moment matrix $\mathcal{M}(n)(\beta)$ to be positive semidefinite, and for the algebraic variety associated to β, $\mathcal{V} \equiv \mathcal{V}_\beta$, to satisfy $\operatorname{rank} \mathcal{M}(n) \leq \operatorname{card} \mathcal{V}$ as well as the following *consistency* condition: if a polynomial $p(x) \equiv \sum_{|i| \leq 2n} a_i x^i$ vanishes on $\mathcal{V}$, then $p(\beta) := \sum_{|i| \leq 2n} a_i \beta_i = 0$. In joint work with Lawrence Fialkow and Michael Möller, we employ tools and techniques from algebraic geometry (e.g., Hilbert polynomials, Gröbner and H-bases, representation of positive polynomials) to prove that for the *extremal* case ($\operatorname{rank} \mathcal{M}(n) = \operatorname{card} \mathcal{V}$), positivity of $\mathcal{M}(n)$ and consistency are sufficient for the existence of a (unique, $\operatorname{rank} \mathcal{M}(n)$-atomic) representing measure.

Truncated moment problems (TMP) as above for which the support of a representing measure is required to lie inside a closed set K are called truncated K-moment problems (TKMP). In case K is a semi-algebraic set determined by polynomials q_1,...,q_m, the study of TKMP is dual to determining whether a polynomial nonnegative on K belongs to the positive cone consisting of polynomials of degree at most $2n$ which can be expressed as sums of squares, and of squares multiplied by one or more distinct q_i's.

The extremal case, which we have now solved, is inherent in the TMP. A recent result of C. Bayer and J. Teichmann (extending a classical theorem of V. Tchakaloff and its successive generalizations given by I.P. Mysovskikh, M. Putinar, and L. Fialkow and the speaker) implies that if $\beta^{(2n)}$ has a representing measure, then it has a finitely atomic representing measure. In joint work with L. Fialkow, we had previously shown that $\beta^{(2n)}$ has a finitely atomic representing measure if and only if $\mathcal{M}(n) \equiv \mathcal{M}(n)(\beta)$ admits an extension to a positive moment matrix $\mathcal{M}(n+k)$ (for some $k \geq 0$), which in turn admits a rank-preserving (i.e., *flat*) moment matrix extension $\mathcal{M}(n+k+1)$. Further, we proved that any flat extension $\mathcal{M}(n+k+1)$ is an extremal moment matrix for which there is a computable rank $\mathcal{M}(n+k)$-atomic representing measure μ. In this sense, the existence of a representing measure for $\beta^{(2n)}$ is intimately related to the solution of an extremal TMP.

Representation of reduced special groups in algebras of continuous functions

Max Dickmann, University of Paris VII, France

(Joint work with Francisco Miraglia)

In this talk we address Marshall's long standing *representation problem* for abstract order spaces from a new perspective. Marshall's question is whether any abstract space of orderings is isomorphic (in the category of such spaces) to the space of orderings of a (orderable) Pythagorean field; for background information see [M]. This problem - which admits several meaningful variants - has remained open for over thirty years. There is a prevailing feeling that the answer may well be negative; some even suspect that the statement may be independent of the axioms of set theory. We use here the dual formulation in terms of *reduced special groups* (RSG), cf. [DM].

We show that, upon (slightly?) broadening the class of representation objects from the RSGs of orderable, Pythagorean fields to the RSGs associated with certain rings, *the problem has a positive answer.* More precisely, we show that any RSG is isomorphic to the RSG associated to some multiplicative group of invertible, continuous real-valued functions on a Boolean space X. In particular, the underlying ring, $C(X)$, is a Pythagorean ring with many units which, in addition, is a real closed ring in the sense of Prestel-Schwartz [PS].

References:

[DM] M. Dickmann, F.Miraglia, *Special Groups : Boolean-Theoretic Methods in the Theory of Quadratic Forms*, Memoirs Amer. Math. Soc. 689, Providence, R.I., 2000.

[M] M. Marshall, *Spaces of Orderings and Abstract Real Spectra* , Lecture Notes in Mathematics 1636, Springer-Verlag, Berlin, 1996.

[PS] A. Prestel, N. Schwartz, *Model Theory of Real Closed Rings*, Fields Institute Comm. 32 (2002), 261-290.

Zero patterns and unitary similarity

Dragomir Djoković, University of Waterloo, Canada

(Joint work with Jinpeng An)

We specify a subspace of $n \times n$ complex matrices by requiring certain entries to vanish. We refer to the collection of the positions of these entries as a zero pattern. We also impose the technical conditions that the zero pattern contains no diagonal entries and that the total number of required zero entries is $n(n-1)/2$. A classic example of such a zero pattern is the lower (or upper) triangular pattern consisting of all entries below (above) the diagonal. Schur's theorem (which is almost 100 years old) asserts that the lower (or upper) zero pattern is universal in the sense that every $n \times n$ complex matrix is unitarily similar to one that admits this zero pattern.

In this talk I shall describe new results proving the existence of many new universal zero patterns. We shall also survey what is known in the cases when n is less than or equal to 4. If time permits, we shall also mention the real and quaternionic versions of our results.

Our paper containing the proofs of the most important results (in a more general setup) is available on arXiv.

An abstract Fejer-Riesz theorem

Michael A. Dritschel, University of Newcastle, United Kingdom

The classical Fejer-Riesz theorem states that a nonnegative complex trigonometric polynomial can be written as the square of an analytic trigonometric polynomial, and that this may be chosen to be outer. Marvin Rosenblum proved a version of this theorem for operator valued polynomials, and the speaker showed that factorization as a sum of squares of analytic polynomials held for strictly positive polynomials in the multivariate case (the question of outer factorizations a separate issue addressed together with Hugo Woerdeman). In this talk we consider Fejer-Riesz type factorizations over a (noncommutative) algebra with an archimedean quadratic module, replacing the commutative torus groups by an ordered group.

Operators determining the norm

José Extremera Lizana, University of Granada, Spain

Let G be a locally compact abelian group, let X be a Banach space and let τ be a strongly continuous bounded representation of G on X. Let $\mathfrak{A}(\tau)$ be the weakly closed subalgebra of $L(X)$ generated by all Fourier transforms $\int_G \tau(t)d\mu(t)$ with $\mu \in M(G)$. We say that $T \in \mathfrak{A}(\tau)$ determines the norm of X if every continuous norm $\|\cdot\|$ on X making the operator $T : (X, \|\cdot\|) \to (X, \|\cdot\|)$ continuous is equivalent to the Banach space norm of X. In this paper we study neccesary and sufficient conditions on T in order to determine the norm of X. Such conditions are mainly related with the topological structure of the Arveson spectrum of T and with the existence of critical eigenvalue of T.

Molecular conduction and the characteristic polynomial

Patrick W. Fowler, University of Sheffield, United Kingdom

(Joint work with Barry T. Pickup and Tsanka Z. Todorova)

The standard Hückel theory of conjugated hydrocarbons is equivalent to the problem of determining the spectrum of the adjacency matrix of the molecular graph. Conduction of an electron through a molecular electronic device is modelled by an extension of this theory. For conduction we solve the (now continuous) eigenvalue problem under the modified source-and-sink boundary conditions used by Ernzerhof and his group. It is found that the conductance varies strongly with the eigenvalue (representing the energy of the ballistic electron) but, within the approximations of Hückel theory, this property is fully determined by a combination of four characteristic polynomials: those of the molecular graph and three vertex-deleted sub-graphs. Trends and systematic descriptions of the physics to be expected from prototype molecular devices follow from this analytical expression. In particular, closed-form expressions and properties of the conductance function are deduced for some classes of chemical graphs. Relations of these ballistic currents to the 'ring currents' induced in aromatic molecules by magnetic fields will also be discussed.

Keywords: chemical graphs; adjacency matrices; eigenvalues; characteristic polynomials

The pp conjecture in the theory of spaces of orderings

Pawel Gladki, University of California, Santa Barbara, USA

Spaces of orderings were introduced by Murray Marshall in the 1970's and provide an abstract framework for studying orderings on fields and the reduced theory of quadratic forms over fields. Numerous important notions in this theory, such as isometry, isotropy, or being an element of a value set of a form, make an extensive use of positive primitive formulae in the language of special groups. Therefore, the following question, which can be viewed as a type of very general local-global principle, is of great importance: is it true that if a positive primitive formula holds in every finite subspace of a space of orderings, then it also holds in the whole space? This problem is now known as the pp conjecture. The answer to this question is affirmative in many cases, although it has always seemed unlikely that the conjecture has a positive solution in general. In this talk we shall discuss first counterexamples for which the pp conjecture fails – namely, we shall classify spaces of orderings of function fields of rational conics with respect to the pp conjecture, and show for which of such spaces the conjecture fails, and then discuss the pp conjecture for the space of orderings of the field $\mathbb{R}(x, y)$. The mentioned results are extracted from author's dissertation.

Real holomorphy rings and the complete real spectrum

Danielle Gondard-Cosette, University Paris VI, France

(Joint work with Murray Mashall)

After recalling known facts on the real holomorphy ring of a field, we present possible corresponding definitions for a ring, and then deal with what we called "Real Holomorphy Rings" and "Complete Real Holomorphy Rings". Then we introduce and study the new notion of "Complete Real Spectrum".

Matrix length

Alexander Guterman, Moscow State University, Russia

(Joint work with Olga Markova)

We investigate the length function on associative algebras and its algebraic properties. In particular, we study the length of direct sums, homomorphic images and subalgebras and the behavior of the length under the adjunction of unity and the field extensions. Also we estimate the length of local algebras via the nilpotency index of its Jacobson radical. Length of commutative matrix algebras is investigated.

Diameter preserving surjections in the geometry of matrices

Hans Havlicek, Vienna Technical University, Austria

(Joint work with Wen-ling Huang (Hamburg))

We consider a class of graphs subject to certain restrictions, including the finiteness of diameters. Any surjective mapping $\varphi : \Gamma \to \Gamma'$ between graphs from this class is shown to be an isomorphism provided that the following holds: Any two points of Γ are at a distance equal to the diameter of Γ if, and only if, their images are at a distance equal to the diameter of Γ'.

This result is then applied to the graphs arising from the adjacency relations of spaces of rectangular matrices, spaces of Hermitian matrices, and Grassmann spaces (projective spaces of rectangular matrices).

Keywords. Adjacency preserving mapping, diameter preserving mapping, geometry of matrices, Grassmann space.

MSC: 51A50, 15A57.

Interlaced eigenvalues and quantum information theory

John Holbrook, University of Guelph, Canada

The rank-k numerical range of a matrix M is the set of complex z such that for some rank-k projection P we have $PMP = zP$. Among the many generalizations that have been proposed for the classical numerical range (which is the rank-1 numerical range), this one seems especially promising. It has, for example, applications in QIT (quantum information theory); indeed, its study was first suggested by problems in quantum error correction. It also provides a striking extension of the Toeplitz-Hausdorff theorem: numerical ranges of all ranks are convex subsets of the complex plane. In this talk we'll survey recent developments, including the connections with certain eigenvalue interlacing phenomena that have a history stretching all the way back to Cauchy.

Nilpotent subalgebra of the commutator

Anthony Iarrobino, Northeastern University, USA

(Joint work with R. Basili)

Let $Mat_n(K)$ denote the ring of $n \times n$ matrices over a field K. Fix a nilpotent $n \times n$ matrix B of Jordan partition P, and consider the commutator algebra $C_B \subset Mat_n(K)$ of B, and its subset N_B of nilpotent matrices. R. Basilli defined a certain maximal commutative nilpotent subalgebra $N = N_{B,sp}$ of N_B. We discuss natural bases for the quotients N^i/N^{i+1}. We also pose some questions about the connection with the problem of determining the Jordan partition $Q(P)$ of a generic element of N_B.

When Jordan submodules are bimodules

Edward Kissin, London Metropolitan University, United Kingdom

(Joint work with Matej Brešar and Victor Shulman)

Let $\mathcal{A}$ be an associative algebra and let X be an $\mathcal{A}$-bimodule. We call a linear subspace Y of X a Jordan $\mathcal{A}$-submodule of X if $Ay + yA \in Y$ for all $A \in \mathcal{A}$ and $y \in Y$ (if $X = \mathcal{A}$, then this coincides with the classical concept of a Jordan ideal). When is a Jordan $\mathcal{A}$-submodule a submodule? We give a thorough analysis of this question in both algebraic and analytic context. We consider general algebras and general Banach algebras and treat some more specific topics, such as symmetrically normed Jordan $\mathcal{A}$-submodules. Some of our results are of interest also in the classical situation; in particular, we show that there exist C^*-algebras having Jordan ideals that are not ideals.

Norm inequalities for commutators of self-adjoint operators

Fuad Kittaneh, University of Jordan, Jordan

Let A, B, and X be bounded linear operators on a complex separable Hilbert space. It is shown that if A and B are self-adjoint with $a_1 \leq A \leq a_2$ and $b_1 \leq B \leq b_2$ for some real numbers a_1, a_2, b_1, and b_2, then for every unitarily invariant norm $|||\cdot|||$,

$$|||AX - XB||| \leq \max(a_2 - b_1, b_2 - a_1)\, |||X|||\,.$$

If, in addition, X is positive, then

$$|||AX - XA||| \leq \frac{1}{2}(a_2 - a_1)\, |||X \oplus X|||\,.$$

These norm inequalities generalize recent related inequalities due to Kittaneh, Bhatia-Kittaneh, and Wang-Du.

Sums of hermitian squares, the BMV conjecture and Connes' embedding problem

Igor Klep, University of Ljubljana, Slovenia

(Joint work with Markus Schweighofer)

We consider polynomials in noncommuting symmetric variables. Which of these polynomials yield a matrix with positive trace whenever matrices are substituted for the variables? Can one find algebraic certificates for the global positivity of the trace? Natural candidates involve sums of hermitian squares and can be searched for using semidefinite programming. We relate this to two long-standing and famous conjectures: the Bessis-Moussa-Villani conjecture from quantum physics and Connes' embedding problem for type II_1 von Neumann algebras.

On operator hyperreflexivity of subspace lattices

Kamila Kliś-Garlicka, Agricultural University of Krakow, Poland

Let $\mathcal{H}$ denote a Hilbert space and $\mathcal{P}(\mathcal{H})$ a lattice of all orthogonal projections on $\mathcal{H}$.

A subspace lattice $\mathcal{L}$ is called *operator hyperreflexive* if there is a constant $C > 0$ such that

$$\operatorname{dist}(P, \mathcal{L}) \leq C \sup_{\|x\| \leq 1} \operatorname{dist}(Px, \mathcal{L}x)$$

for all projections $P \in \mathcal{P}(\mathcal{H})$.

Some results and examples will be presented. In particular we will show that all orthogonal complemented CSL are operator hyperreflexive.

Supertropical semirings and supervaluations

Manfred Knebusch, University of Regensburg, Germany

(Joint work with Louis Rowen and Zur Izhakian)

It has long been felt that along with the now rapidly evolving tropical geometry there should be developed a fitting tropical algebra. I will report on a new approach to tropical algebra, still very much work in progress with Louis Rowen and Zur Izhakian, both at Bar Ilan University, Tel Aviv. In contrast to established tropical algebra with its max-plus semirings, we build and study "supertropical" semirings. Here the sum of two elements, which in the max-plus setting would be zero, is a "ghost" element. Intuitively a ghost element is near to zero, but not exactly zero. Every "tangible" element of the semiring has an associated ghost. Usually one obtains results, e.g identities, which are only true up to ghost elements. This is the prize we have to pay for the fact, that we have no subtraction in semirings. The prize is definitely lower than in usual tropical algebra. The supertropical approach seems to be particulary apt to deal with valuations. To any valuation on a ring R (in the usual Bourbaki sense) we associate a "supervaluation" on R, which takes values in a supertropical semiring. This allows us to ask new questions about valuations, already in the case of fields. If time allows, I will present a result on convex valuations on a ring, which can be stated completely within usual real algebra, and probably has escaped the attention of the not supertropically minded real algebraists.

Matrix inequalities and convex functions

Tomaž Kosem, University of Ljubljana, Slovenia

Many matrix inequalities can be derived from scalar inequalities for convex functions. Using the functional calculus scalars can be replaced with Hermitian matrices and appropriate matrix quantities can be compared, e.g. unitarily invariant norms, eigenvalues etc. Also, the Loewner partial order frequently takes place in such inequalities.

Besides some operator versions of Jensen's inequality some results are presented, which are connected with subhomogeneity and subadditivity of concave functions. A generalization of Ando-Zhan inequality between $|||f(A)+f(B)|||$ and $|||f(A+B)|||$ for convex/concave function is shown. Finally, Young's inequality $ab \leq \Phi(a) + \Psi(b)$ and its old and novel matrix interpretations are treated.

A matrix approach to generalized Petersen graphs

Istvan Kovacs, University of Primorska, Slovenia

(Joint work with A. Malnič, D. Marušič and Š. Miklavič)

Let Γ be a simple finite graph with an automorphism permuting the vertices into cycles of the same length. In the talk we study the automorphism group of Γ via spectral properties of Γ. In particular, we give a new and short proof of an old result of Frucht, Graver and Watkins (*Proc. Camb. Phil. Soc.*, **70** (1971), 211-218) classifying edge-transitive generalized Petersen graphs, and also discuss possible generalizations.

Combinatorial Gelfand models for some semigroups and q-rook monoid algebras

Ganna Kudryavtseva, Kyiv Taras Shevchenko University, Ukraine

We construct combinatorial Gelfand models for semigroup algebras of some finite semigroups, which include the symmetric inverse semigroup, the dual symmetric inverse semigroup, the maximal factorizable subsemigroup in the dual symmetric inverse semigroup, and the factor power of the symmetric group. Furthermore we extend the Gelfand model for the semigroup algebras of the symmetric inverse semigroup to a Gelfand model for the q-rook monoid algebra.

Positive polynomials on projective limits of real algberaic varieties

Salma Kuhlmann, University of Saskatchewan, Canada

(Joint work with M. Putinar)

We prove a Positivstellensatz on the fibre product of real algebraic affine varieties, and iterate it to a comprehensive class of projective limits of such varieties. This framework includes as necessary ingredients recent works on the multivariate moment problem, disintegration and projective limits of probability measures and basic techniques of the theory of locally convex vector spaces.

Some new bounds on realizing spectra in the nonnegative inverse eigenvalue problem

Tom Laffey, University College Dublin, Ireland

(Joint work with Helena Šmigoc)

Let $\sigma := (\lambda_1, \quad \ldots \quad, \lambda_n)$ be a list of complex numbers. The nonnegative inverse eigenvalue problem (NIEP) asks for necesssary and sufficient conditions on σ to be the spectrum of a matrix with nonnegative real entries. We will summarize the literature on this problem and present some new sufficient results. Suppose that

$$\text{maximum}\{| \lambda_j | \quad : \quad j = 1, 2, \quad \ldots \quad n\}$$

is a non-repeated element of σ. Let

$$s_k := \lambda_1^k + \quad \ldots \quad + \lambda_n^k, \quad k = 1, 2, 3, \ldots.$$

be the associated Newton power sums. Then a celebrated result of Boyle and Handelman states that if all the s_k are positive, then there exists a nonnegative integer N such that

$$\sigma_N := (\lambda_1, \quad \ldots \quad, \lambda_n, 0, \quad \ldots \quad, 0), \qquad (N \text{ zeros})$$

is the spectrum of a nonnegative $(n+N)\times(n+N)$ matrix. The problem of obtaining a constructive proof of this result with an effective bound on the minimum number N of zeros required has not yet been solved. We will discuss an approach to this using polynomials, and apply it to the test spectrum $(3+t, 3-t, -2, -2, -2)$. In particular, we will show that, for $t > 0$, if

$$N > 2\log_3(\frac{2}{t}),$$

then

$$(3+t, 3-t, -2, -2, -2, 0, \quad \ldots \quad, 0), \qquad (N \text{ zeros})$$

is the spectrum of a nonnegative $(n+N)\times(n+N)$ matrix.

Real solving polynomial equations with semidefinite programming

Monique Laurent, Center for Mathematics and Computer Science, Netherlands

(Joint work with J.B. Lasserre and P. Rostalski)

While good methods exist for computing complex roots to polynomial equations, the problem of computing all *real* solutions is less well understood. We propose a numerical method for finding the real solutions to a system of polynomial equations $h_1 = 0, \ldots, h_m = 0$, assuming their number is finite (while the number of complex roots could be infinite). Our method relies on expressing the real radical ideal $\sqrt[\mathbb{R}]{I}$ of the ideal I generated by $h_1, \ldots, h_m$ as the kernel of a suitable positive semidefinite moment matrix. We use semidefinite optimization for finding such a matrix, combined with linear algebra techniques for computing the real roots as well as a (border or Gröbner) basis of the ideal $\sqrt[\mathbb{R}]{I}$.

The same method can be adapted to find the complex roots of I, simply by omitting the positive semidefinite condition, in which case it returns a border base of an ideal J nested between I and its radical $\sqrt{I}$, with $J = I$ precisely when $\mathbb{R}[x]/I$ is a Gorenstein algebra. Using recent ideas from Janovitz-Freireich et al. we can then also find a base of the radical ideal $\sqrt{I}$.

It turns out that our stopping criterion (based on some flatness assumption for moment matrices) is closely related to the stopping criterion used by Zhi and Reid in their algorithm for complex roots inspired by involutive methods for systems of linear PDE's. Moreover, the Zhi-Reid algorithm can be adapted to finding real roots by adding positive semidefinite constraints.

Bratteli diagrams and related groups

Yaroslav Lavrenyuk, Kyiv Taras Shevchenko University, Ukraine

(Joint work with Volodymyr Nekrashevych)

We classify locally finite groups, which are inductive limits of direct products of alternating groups with respect to block-diagonal embeddings. Also for such groups the normal structure is investigated. This class of groups includes a well known class of simple locally finite groups (so-called LDA-groups). We show that two such groups are isomorphic if and only if the AF-algebras defined by the respective Bratteli diagrams are isomorphic. Then the classical results on classification of AF-algebras can be applied.

Recent results on the higher rank numerical range

Chi-Kwong Li, College of William and Mary, USA

The study of higher rank numerical ranges was motivated by quantum error correction. It turns out the higher rank numerical range is also useful in the study of many other topics such as isotropic subspaces, matrix equations, dilation theory, perturbation of operators, matrix inequalities, preserver problems, joint numerical ranges and joint essential numerical ranges, In this talk, some recent results on the joint numerical range and its connections to other subjects will be discussed.

When "nearly" is good enough

Leo Livshits, Colby College, USA

In their 1995 paper Jafarian, Radjavi, Rosenthal and Sourour showed that a collection of compact operators on a Hilbert space is simultaneously triangularizable if and only if it is "nearly commutative". In 2004 Yahagi strengthened the theory by showing that "near-triangularizability" implies triangularizabilty, and that in finite dimensions "near-reducibility" gives reducibility. In this short talk we will demonstrate the corresponding results for collections of (entry-wise) non-negative matrices, from the point of view of "standard triagularizability" (i.e. triangularizability via a simultaneous permutation similarity).

Strongly compact operators

Victor Lomonosov, Kent State University, USA

A linear operator T in a Banach space is strongly compact if the unit ball of the algebra, generated by this operator is compact in the strong operator topology. A linear operator T is essentially normal if its commutator with the dual operator is compact.
Theorem. Suppose that an essentially normal operator T and its dual operator both are not strongly compact. Then the operator T is non-transitive.

Closure and saturation of finitely generated preorderings and quadratic modules of $\mathbb{R}[\underline{x}]$

Murray Marshall, University of Saskatchewan, Canada

(Joint work with T. Netzer, S. Kuhlmann and J. Cimprič)

Let K be a basic closed semialgebraic set of $\mathbb{R}^n$ defined by polynomial inequalities $g_i \geq 0$, $i = 1, \cdots, s$; $\mathrm{Pos}(K) = \{f \in \mathbb{R}[\underline{x}] \mid f \geq 0 \text{ on } K\}$; $T =$ the preordering or quadratic module of $\mathbb{R}[\underline{x}]$ generated by $g_1, \ldots, g_s$; $\overline{T} =$ the closure of T with respect to the unique finest locally convex topology on $\mathbb{R}[\underline{x}]$. In recent years various people have struggled to understand the relationship between $T, \overline{T}$ and $\mathrm{Pos}(K)$. Obviously $T \subseteq \overline{T} \subseteq \mathrm{Pos}(K)$. I will give some of my thoughts on this subject, part of the beginning of a joint project with T. Netzer and J. Cimprič, and will also mention new results in the compact case, part of joint work with S. Kuhlmann and J. Cimprič.

Big bases

Ben Mathes, Colby College, USA

G. Kalisch showed how to construct an operator whose spectrum consists entirely of point spectrum and equals an arbitrarily prescibed compact subset of the plane. We give a new proof of this result and generalize it, revealing its relation to Banach algebras and the theory of strictly cyclic algebras. Our generalization is the following: given any compact subset of Euclidean space, we construct an abelian semisimple strictly cyclic algebra whose Gelfand spectrum is homeomorphic to that compact set.

The sum of largest eigenvalues of graphs and matrices

Bojan Mohar, Simon Fraser University, Canada

Gernert conjectured that the sum of two largest eigenvalues of the adjacency matrix of a simple graph of order n is at most n. This can be proved, in particular, for all regular graphs. Gernert's conjecture was disproved by Nikiforov, who also provided a nontrivial upper bound for the sum of two largest eigenvalues. We will present improvements and extensions of these results to general symmetric $n \times n$ matrices and discuss extremal cases. Other recent results on the extreme behavior of the sum the k largest eigenvalues of symmetric matrices and, in particular, adjacency matrices of graphs will also be presented.

Blum-Hanson property and quasisimilarity of operators

Vladimir Müller, Czech Academy of Sciences, Czech Republic

Let T be a contraction on a Hilbert space H such that T^n converges in the weak operator theory. By a result motivated by ergodic theory then T has the Blum-Hanson property, i.e., $\lim_{N\to\infty} N^{-1}\sum_{n=1}^{N} T^{k_n}x$ exists in the norm topology for each $x \in H$ and each increasing subsequence (k_n). We show that this is not true for power bounded Hilbert space operators. This also implies that there are power bounded operators which are not quasisimilar to a contraction.

This answers an open problem from ergodic theory as well as questions concerning (quasi)similarity of operators.

New series of strongly regular graphs

Mikhail Muzychuk, Netanya Academic College, Israel

In 1971 W.D. Wallis proposed a new construction of strongly regular graphs based on an affine design and a Steiner 2-design. Thirty years later D.G. Fon-Der-Flaass found how to introduce a sort of randomness into Wallis construction. He built a hyperexponentially many strongly regular graphs with the same parameters, but his construction covered only one case of Wallis construction, namely when the corresponding Steiner design has block size 2. The goal of this talk is twofold. First, I show how to modify Fon-Der-Flaass ideas in order to cover all the cases of Wallis construction. Second, it will be shown that a Steiner 2-design in the original Wallis construction may be replaced by a partial linear space with some additional properties. As a result new constructions of strongly regular graphs will be presented.

Linear transport problems in networks

Reiner Nagel, University of Tübingen, Germany

We study the linear Boltzmann equation in a network combining methods from graph theory and functional analysis/operator theory. Our main interest is in the qualitative behavior and it is shown that - under appropriate assumptions on the transport process and on the structure of the graph - there occurs asymptotic periodicity. The results are due to Marjeta Kramar Fijavž, Eszter Sikolya and others. More details and open problems will be discussed in the working group Dynamical Networks.

On the sequential closure of quadratic modules

Tim Netzer, University of Konstanz, Germany

We examine the sequential closure of a quadratic module M. It consists of all elements f for which there is some element q, such that $f + \epsilon q \in M$ for all $\epsilon > 0$. We give a sufficient condition for an element to belong to this sequential closure. The condition involves a family of "lower dimensional" quadratic modules, corresponding to fibres of bounded polynomial functions. We apply the result to certain families of quadratic modules in the polynomial ring.

Products and sums of quasi-nilpotent operators

Nika Novak, University of Ljubljana, Slovenia

We will give an overview of the results concerning products and sums of nilpotent operators, square-zero operators and quasi-nilpotent operators. We will consider the operators on a finite and infinite-dimensional Hilbert space. In each case, we will be mainly concerned with the characterization of products and sums of two operators.

On commuting nilpotent matrices

Polona Oblak, University of Ljubljana, Slovenia

For a nilpotent matrix B, we discuss the set $\mathcal{P}(\mathcal{N}_B)$ of all Jordan canonical forms of matrices in the nilpotent commutator of matrix B.

We give an overview of some recent results and for certain special cases of B, we give the complete lists of $\mathcal{P}(\mathcal{N}_B)$.

Faithful linear representations of bands

Jan Okniński, Warsaw University, Poland

(Joint work with Ferran Cedo)

A semigroup S such that $a^2 = a$ for every $a \in S$ is called a band. The main motivating problem for this talk is to find conditions on a band S in order that S embeds into the multiplicative semigroup $M_n(F)$ of $n \times n$ matrices over a field F for some $n \geq 1$. It is known and easy to show that this is always the case if S is a rectangular band (that is, a semigroup satisfying the identity $xyx = x$), but this is no longer true in general (that is, in case the band S has at least 2 rectangular band components). The following related problem will be also discussed: when the semigroup algebra $K[S]$ of a band S over a field K is embeddable into $M_n(A)$ for a commutative algebra A? Certain general results will be proved and some concrete embeddings will be constructed.

Linear automata and residually finite groups

Andriy Oliynyk, Kyiv Taras Shevchenko University, Ukraine

Linear automata are transducers whose output and transition functions are linear mappings. Groups generated by linear automata admits simple algebraic description. These groups are metabelian. We present possible realizations of residually finite metabelian groups as groups generated by linear automata. Connections with infinite dimensional unitriangular groups will be discussed as well.

Matrix constructions of finite metric spaces

Bogdana Oliynyk, National University Kyiv-Mohyla Academy, Ukraine

Two metric spaces are called isomorphic if there exists a one-to-one mapping between them that preserves equalities and strict inequalities of distances in these spaces. Each finite metric space is uniquely defined by its matrix of distances as elements of this space are enumerated. We consider some constructions of finite metric spaces up to isomorphism. The matrices of distances of this constructions are characterized.

Additive minimal–rank nonincreasing maps on ✠-hermitian/✠-skew-hermitian matrices

Marko Orel, University of Ljubljana, Slovenia

(Joint work with B. Kuzma)

Let $\mathbb{K}$ be a field, $\mathcal{M}_n(\mathbb{K})$ the algebra of all n–by–n matrices with entries from $\mathbb{K}$, and $✠ : \mathcal{M}_n(\mathbb{K}) \to \mathcal{M}_n(\mathbb{K})$ an involution. A matrix $A \in \mathcal{M}_n(\mathbb{K})$ is *✠-hermitian* (resp. *✠-skew-hermitian*) if $✠(A) := A^{✠} = A$ (resp. $A^{✠} = -A$). The set of all such matrices is denoted by $\mathcal{H}_n^{✠}(\mathbb{K})$ (resp. $\mathcal{SH}_n^{✠}(\mathbb{K})$). The minimal (nonzero) rank is defined as $r_{min} := \min\{\operatorname{rk} A \,|\, 0 \neq A \in \mathcal{H}_n^{✠}(\mathbb{K})\}$ (resp. $r_{min} := \min\{\operatorname{rk} A \,|\, 0 \neq A \in \mathcal{SH}_n^{✠}(\mathbb{K})\}$). A map Φ is *minimal–rank nonincreasing* if for any A, $\operatorname{rk} A = r_{min}$ implies $\operatorname{rk} \Phi(A) \leq r_{min}$. The classification of all such additive maps on $\mathcal{H}_n^{✠}(\mathbb{K})$ (resp. $\mathcal{SH}_n^{✠}(\mathbb{K})$) will be presented.

Spaces of real places

Katarzyna Osiak, University of Silesia, Poland

An $\mathbb{R}$ - place of a formally real field K is a place $\xi : K \longrightarrow \mathbb{R} \cup \{\infty\}$. The set of all $\mathbb{R}$-places of the field K is denoted by $M(K)$. Every $\mathbb{R}$ - place of K is connected with some subset of the space $X(K)$ of orderings of the field K. Namely, if ξ is an $\mathbb{R}$ - place, then there exists an ordering P such that the set

$$A(P) := \{a \in K : \exists q \in \mathbb{Q}^+ (q \pm a \in P)\}$$

is the valuation ring of ξ. We say that P determines ξ in this case. Any ordering P of the field K determines exactly one $\mathbb{R}$ - place.

The above described correspondence between orderings and $\mathbb{R}$-places defines a surjective map

$$\lambda_K : \mathcal{X}(K) \longrightarrow M(K),$$

which, in turn, allows us to equip $M(K)$ with the quotient topology inherited from $\mathcal{X}(K)$. $M(K)$ is a Hausdorff space. It is also compact as a continuous image of a compact space. But the problem

Which compact, Hausdorff spaces occur as the spaces of real places?

is still open.

We prove that:

- every Boolean space is a space of $\mathbb{R}$ - places of some formally real field K;
- the segment [0,1] is realised as a space of $\mathbb{R}$ - places for some algebraic extension of $\mathbb{R}(X)$;
- the space of $\mathbb{R}$ - places of a function field over any real closure of $\mathbb{R}(X)$ is not a metrizable space and therefore $M(\mathbb{R}(X,Y))$ is not metrizable.

On the irreducibility of commuting varieties

Dmitri Panyushev, Independent University of Moscow, Russia

In my talk, I am going to review the irreducibility problem for various types of "commuting varieties". The basic example is the variety $C = C(2,n)$ of pairs of commuting n by n complex matrices. It is well-known that $C(2,n)$ is irreducible. There are several directions for generalising this situation. One can consider

1) arbitrary reductive Lie algebras in place of the matrix algebra $gl(n)$;
2) commuting varieties associated with non-reductive subalgebras of $gl(n)$;
3) triples, quadruples, etc. of matrices, i.e., varieties $C(3,n)$, etc.
4) intersection of C with subspaces of $gl(n) \times gl(n)$;
5) subvarieties of C determined by natural polynomial conditions.

In many cases, the varieties obtained appear to be reducible, and the natural problem is to describe their irreducible components. We also discuss some open problems related to commuting varieties of matrices.

On the max version of the generalized spectral radius theorem

Aljoša Peperko, University of Ljubljana, Slovenia

Let Ψ be a bounded set of $n \times n$ non-negative matrices. Recently, the max algebra version $\mu(\Psi)$ of the generalized spectral radius of Ψ was introduced. We show that

$$\mu(\Psi) = \lim_{t\to\infty} \rho(\Psi^{(t)})^{1/t},$$

where ρ denotes the generalized spectral radius and $\Psi^{(t)}$ the Hadamard power of Ψ. This provides a description of $\mu(\Psi)$ that uses no max terminology. As an application we give a short proof of the max version of the generalized spectral radius theorem.

MSC(2000): 15A18, 15A48, 15A60
Key words: Maximum circuit geometric mean, Max algebra, Generalized spectral radius, Joint spectral radius, Hadamard powers, Schur powers.

The moment problem for two-dimensional semialgebraic sets

Daniel Plaumann, University of Konstanz, Germany

Let h be a polynomial in n variables, and let $S = \{x \in \mathbb{R}^n : h(x) \geq 0\}$. The moment problem for the set S roughly translates into the question whether every polynomial that is positive on S can be approximated by polynomials of the form $s + th$ where s and t are sums of squares of polynomials. We discuss various techniques to approach this problem, in particular in the case $n = 2$.

Algebraic characterizations of operator algebras via $*$-double construction

Stanislav Popovych, Chalmers University of Technology, Sweden

A characterization of the $*$-subalgebras of the algebra of bounded operators on Hilbert space is presented. It is analogues to Choi and Effros characterization of abstract operator systems. Sufficient conditions for the O^*-representability of a $*$-algebra in terms of its Greobner basis are given. These conditions are generalization of the unshrinkability of monomial $*$-algebras introduced by C. Lance and P. Tapper. Characterization of non-self-adjoint subalgebras of $B(H)$ is given via $*$-double functor.

Pólya's theorem with zeros

Victoria Powers, Emory University, USA

Let $\mathbb{R}[X] := \mathbb{R}[X_1, \ldots, X_n]$. Pólya's Theorem says that if a form (homogeneous polynomial) $p \in \mathbb{R}[X]$ is positive on the standard n-simplex Δ_n, then for sufficiently large N all the coefficients of $(X_1 + \cdots + X_n)^N p$ are positive. In 2001, Powers and Reznick gave a bound on the N needed, in terms of the degree of p, the coefficients, and minimum of p on Δ_n. This quantitative Pólya's Theorem has many applications, in both pure and applied mathematics. The work is this talk is part of an ongoing project to understand when Pólya's Theorem holds for forms if the condition "positive on Δ_n" is relaxed to "nonnegative on Δ_n", and to give bounds on the N. We prove a "localized" Pólya's Theorem, with a bound on the N needed, which is a quantitative version of a result of Schweighofer. We use this result to give a sufficient condition for forms which are non-negative on Δ_n to satisfy the conclusion of Pólya's Theorem, with a bound on the N needed.

Archimedean quadratic modules of real polynomials

Alexander Prestel, University of Konstanz, Germany

The polynomials $h_1, \dots h_s \in \mathbb{R}[X_1, \dots X_n]$ define an archimedean quadratic module $M(h)$ iff the semialgebraic set $W(h) \subseteq \mathbb{R}^n$ defined by $h_i(x) \geq 0$ for $1 \leq i \leq s$ is bounded and every polynomial $f \in \mathbb{R}[X_1, \dots, X_N]$, strictly positive on $W(h)$, admits a representation

$$f = \sigma_0 + h_1\sigma_2 + \cdots + h_s\sigma_s$$

with σ_i being sums of squares of polynomials. If all h_i are linear and $W(h)$ is bounded, them $M(h)$ is archimedean. But not all bounded $W(h)$ have archimedean $M(h)$.

There exists an abstract valuation theoretic criterion for $M(h)$ to be archimedean. We are, however, concerned with an effective procedures that allows to decide whether $M(h)$ is archimedean or not. In dimension 2 a geometric such procedure was given by E. Cabral in her PhD thesis. In dimension $n \geq 3$ decidability has now been proved by S. Wagner, but a geometric procedure is still lacking.

Some problems raised by Horn's theorem

João Queiró, University of Coimbra, Portugal

Horn's 1962 conjecture on the inequalities describing the possible eigenvalues of the sum of two Hermitian matrices in terms of the eigenvalues of the summands has been proved almost ten years ago. This celebrated result is a consequence of work done by A. Klyachko and A. Knutson + T. Tao. The theorem raises some additional interesting problems. We shall describe two of these: partial spectra description, and effective construction of solutions in the existence part of the theorem.

Local-to-global properties of semigroups of matrices

Heydar Radjavi, University of Waterloo, Canada

By this is meant proving something about a whole semigroup S with available knowledge of "part" of the semigroup. One example is: if every pair of members of S is triangularizable, then S is triangularizable. A different example was given recently by Rosenthal-Radjavi: if a fixed nonzero linear functional applied to the members of S yields only a finite number of values, then S itself is finite. A third result of this sort I proved is: if a rank-one linear functional yields only real values when applied to (a complex S), then S is simultaneously similar to a semigroup of real matrices (and similar results for other fields and subfields).

Perturbations of canonical forms

Leiba Rodman, College of William and Mary, USA

The talk will address various aspects of the local behavior of canonical forms of matrices, matrix pairs etc., under small changes in the matrix. Basic canonical forms are considered, such as the Jordan canonical form, and other forms. Open problems will be stated.

Here is one problem of the type indicated: We say that complex matrices A and B have the same Jordan structure if the number of distinct eigenvalues $\mu_1, \ldots, \mu_k$ of B is equal to the number of distinct eigenvalues $\lambda_1, \ldots, \lambda_k$ of A, and there is a bijection α on $\{1, 2, \ldots k\}$ such that the sequence of partial multiplicities (sizes of Jordan blocks) of A corresponding to λ_j coincides with that of B corresponding to $\mu_{\alpha(j)}$. Given a matrix A, what are the possible Jordan structures such that for every $\varepsilon > 0$ there is a matrix B having the Jordan structure in question and satisfying $\|A - B\| < \varepsilon$? The answer was given by Marcus - Parilis (1983), den Boer - Thijsse (1980).

An anologous problem was solved by Gracia, de Hoyos, Zaballa (1989) for pairs of matrices (A_1, A_2) where A_1 is $n \times n$ and A_2 is $n \times p$, under the group action

$$(A_1, A_2) \quad \longrightarrow \quad (PA_1P^{-1} + PA_2R, PA_2Q^{-1})$$

with invertible P and Q, and assuming that the pair (A_1, A_2) is controllable. The canonical form for this action is known as Brunovsky form (1970).

On polynomial numerical hulls of matrices

Abbas Salemi, University of Kerman, Iran

Let M_n be the set of $n \times n$ complex matrices. For any $A \in M_n$, we use the joint numerical range $W(A, A^2, \ldots, A^l)$ to study the *polynomial numerical hull of order* l of A, denoted by $V^l(A)$. By considering the k-rank joint numerical hulls $\Lambda_k(A, A^2, \ldots, A^l)$, we introduce the *k-rank polynomial numerical hull of order* l of a matrix A, which is defined and denoted by

$$V_k^l(A) = \{\lambda \in \Lambda_k(A) : (\lambda, \lambda^2, \ldots, \lambda^l) \in \text{conv}\,[\Lambda_k(A, A^2, \ldots, A^l)]\}.$$

We study some relationship between these two notations.

Key words: higher rank numerical range, joint numerical range, polynomial numerical hull.

AMS Subject Classification: 15A60, 15A18.

Induced representations and positivity in *-algebras

Yuriy Savchuk, Planck Institute for Mathematics in the Sciences, Germany

For general rings $B \subset A$ and a B-module V the induced A-module is defined as $A \otimes_B V$. We define induced Hilbert space representations in case when A and B are $*$-algebras, i.e. we define a scalar product on $A \otimes_B V$. Thereby the hermitian B-module V should be positive with respect to the quadratic module $\sum A^2 \cap B$. For a special class of algebras we develop Mackey analysis. All notions are illustrated by a number of examples (Weyl algebra, enveloping algebras, q-CCR etc.).

Moment problems for sets of dimension one or two

Claus Scheiderer, University of Konstanz, Germany

Classical moment problems are closely related to functional analysis on one side and to real algebraic geometry on the other. The last years have seen considerable progress in the understanding of this interaction. In the talk I will try to summarize what is known for one- and two-dimensional moment problems, including some results which are not yet published.

Towards noncommutative real algebraic geometry

Konrad Schmüdgen, University of Leipzig, Germany

Artin's solution of Hilbert's famous 17th problem on the representation of positive polynomials as sums of squares of rational functions can be considered as the beginning of real algebraic geometry. In present days Positivstellensaetze on semi-algebraic sets form a central topic of this field. In the talk we shall propose and discuss how basic concepts (positive elements, semi-algebraic sets, quadratic modules, Archimedean orderings) and results (Positivstellensätze) from real algebraic geometry can be generalized to noncommutative $*$-algebras. Various Posititivstellensätze of $*$-algebras (Weyl algebras, enveloping algebras, PI-algebras) will be presented.

The Motzkin polynomial, Putinars quadratic module representation and Connes' embedding problem

Markus Schweighofer, University of Rennes 1, France

(Joint work with Igor Klep)

What happens if the variables in the Motzkin polynomial commute less than they used to? In giving a partial answer to this question, we will show that representation theorems for positive polynomials in commuting variables can have interesting consequences like trace or operator inequalities. We will also include a link to the talks of Igor Klep and Sabine Burgdorf on Connes' embedding problem and the BMV conjecture.

Commuting matrix triples from jet schemes over the commuting pairs variety

Al Sethuraman, California State University, USA

The k-th order jet scheme over a given variety X can be thought of as the set of all parameterized curves of degree k that vanish to degree at least k at some point on X. When X is the variety of commuting pairs of $n \times n$ matrices its k-th order jet scheme can also be interpreted as commuting pairs of $n(k+1) \times n(k+1)$ matrices that also commute with the matrix $J_{k+1} \oplus \cdots \oplus J_{k+1}$ (n summands), where J_{k+1} is the standard nilpotent $k+1 \times k+1$ Jordan block. We describe the structure of a distinguished open set of the k-th order jet scheme of X and explore the irreducibility of the jet scheme for small values of n. We draw inferences for the question of the dimension of the algebra generated by three commuting matrices.

On some lifting problems in C^*-algebras

Tatiana Shulman, Moscow State Aviation Technological University, Russia

For the standard epimorphism from a C^*-algebra A to its quotient A/I by a closed ideal I, one may ask whether an element b in A/I with some specific properties is the image of some element a in A with the same properties. This is known as a lifting problem that can be considered as a non-commutative analogue of extension problems for functions. I am going to discuss some lifting problems connected with the notion of projectivity and semiprojectivity for C^*-algebras, in particular the question about lifting of nilpotent contractions posed by Loring and Pedersen.

Linear operator equations and Beurling-Pollard type theorems

Victor Shulman, Vologda Technical University, Russia

The classical Beurling-Pollard theorem states that a function on the group $\mathbb{R}^n$ or $\mathbb{T}^n$ admits the spectral synthesis, if it is sufficiently smooth or (and) its null set is sufficiently thin. We obtain a version of this result for operator synthesis and apply it to the harmonic analysis in Varopoulos algebras, weighted Fourier algebras and to the study of linear operator equations.

The Portuguese transformation – ten years later

Ilya Spitkovsky, College of William and Mary, USA

The Portuguese transformation is one of a few known approaches to the constructive factorization of triangular almost periodic (AP) matrix functions, arising naturally in applications to convolution type equations on finite intervals. In this talk, we will discuss recent progress made in the almost factorization problem with the use of this transformation. Related open problems will also be presented.

Star complements and strongly regular graphs

Dragan Stevanović, University of Primorska, Slovenia

(Joint work with Marko Milošević)

Star complement of an n-vertex graph G having an eigenvalue λ of multiplicity k is any induced $(n-k)$-vertex subgraph of G which does not have λ as an eigenvalue (of its adjacency matrix). The knowledge of an eigenvalue and its star complement is, in principle, enough to reconstruct the whole graph. We will demonstrate how star complements may be used with strongly regular graphs having eigenvalue 2, in particular, with graphs with parameters $(81, 20, 1, 6)$ and $(105, 32, 4, 12)$.

Low rank perturbations of higher rank numerical ranges

Raymond Nung-Sing Sze, University of Connecticut, USA

For a positive integer k, the rank-k numerical range $\Lambda_k(A)$ of an operator A acting on a Hilbert space $\mathcal{H}$ of dimension at least k is the set of scalars λ such that $PAP = \lambda P$ for some rank k orthogonal projection P.

In this talk, the connection between $\Lambda_k(A)$ and $\Lambda_{k-r}(A+F)$, the rank-$(k-r)$ numerical range of A with a perturbation of a rank r operator F, will be discussed. In particular, it can be shown that if A is normal or if the dimension of A is finite, then $\Lambda_k(A)$ can be obtained as the intersection of $\Lambda_{k-r}(A+F)$ for a collection of rank r operators F.

Furthermore, results for the rank-∞ numerical range $\Lambda_\infty(A)$ will also be studied, where $\Lambda_\infty(A)$ is defined as the set of scalars λ such that $PAP = \lambda P$ for an infinite rank orthogonal projection P.

On varieties of commuting triples

Klemen Šivic, University of Ljubljana, Slovenia

The set $C(3, n)$ of all triples of commuting $n \times n$ matrices over an algebraically closed field F is a variety in F^{3n^2} defined by $3n^2$ equations, which are relations of commutativity. The problem first proposed by Gerstenhaber asks to determine for which natural numbers n this variety is irreducible. This is equivalent to the problem whether $C(3, n)$ equals to the Zariski closure of the subset of all triples of generic matrices (i.e. matrices having n distinct eigenvalues). The answer is known to be positive for $n \leq 7$ and negative for $n \geq 30$. Using simultaneous commutative perturbations of pairs of matrices in the centralizer of the third matrix we prove that $C(3, 8)$ is also irreducible.

Operator space structure of JC*-triples

Richard M. Timoney, Trinity College Dublin, Ireland

In this talk we will concentrate on finite dimensional JC*-triples. As usual, we consider two JC*-triples to be equivalent if there is an isometry from one onto the other, which is the same as an isomorphism of algeraic structures. Our aim is to understand the possible positions in $B(H)$ for JC*-triples equivalent to one fixed JC*-triple X, and it turns out that there is a natural answer in terms of a universal enveloping ternary ring of operators for X. This enveloping ternary ring can be computed for finite dimensional X.

Eigenvalues of sums of selfadjoint matrices

Dan Timotin, Romanian Academy, Romania

(Joint work with H. Bercovici ad W.S. Li)

Suppose we are given three selfadjoint matrices A, B, C, such that $A + B = C$. An old question concerns the determination of the set of possible eigenvalues of C, if the eigenvalues of A and B are given. Horn has conjectured in 1962 that they are characterized by a certain set of inequalities; this very deep conjecture has been proved true in the 90's by work of Klyachko (essentially), Totaro, and Knutson-Tao.

On the other hand, several results had been obtained concerning the location of a single eigenvalue of C. In his survey of the Horn conjecture, published in BAMS in 2000, Fulton has asked the question of the posssible location of a subset of the eigenvalues of C. We give a more general result that completely answers this question, by means of a family of inequalities related to Horn's. As a consequence, a result of Buch (2006) giving conditions for the eigenvalues of Hermitian matrices with positive sum of finite rank is also recaptured.

Non axiomatizability of real spectra and algebraic characterization of completely normal Zariski spectra

Marcus Tressl, University of Manchester, United Kingdom

(Joint work with T. Mellor and N. Schwartz)

Delzell and Madden have constructed a completely normal spectral space that is not the real spectrum of any commutative ring. The property of being completely normal has a natural first order axiomatisation in the Stone dual of the space. I will indicate that it is indeed impossible to characterize spectral subspaces of real spectra in a natural first order way.

Furthermore I will introduce an algebraic invariant of a commutative ring A, which expresses complete normality of the Zariski spectrum of A. Since this invariant is not preserved in ultraproducts (of reduced rings), the property of having completely normal Zariski spectra can not be expressed in the first order theory of (reduced) rings.

Torsion-free Witt groups of hermitian forms

Thomas Unger, University College Dublin, Ireland

(Joint work with Vincent Astier)

It is well-known that the Witt ring of a formally real pythagorean field is torsion free. Let $W(D, \sigma)$ be the Witt group of hermitian forms over a division algebra D with involution σ. I will discuss conditions under which $W(D, \sigma)$ is torsion free.

Linear relations and quotient morphisms

Florian-Horia Vasilescu, University of Lille 1, France

(Joint work with Dana Gheorghe)

Multivalued linear operators, also called linear relations when identified with linear submanifolds of a Cartesian products of two vector spaces, can be associated with linear maps from a vector space, with values in an appropriate quotient space. This remark naturally leads to the concept of quotient morphism, which is a linear map defined between pairs of quotient spaces. In this framework, a generalized composition of quotient morphisms may be defined and (algebraic) Fredholm phenomena may be studied.

Approximately zero-product-preserving maps

Armando Villena, University of Granada, Spain

A continuous linear map T from a Banach algebra A into another B almost preserves the zero products if $\|T(a)T(b)\| \leq \alpha\|a\|\|b\|$ $(a, b \in A)$ for some small positive α. This talk is mainly concerned with the question of whether any continuous linear surjective map $T\colon A \to B$ that almost preserves the zero products is near, with respect to the operator norm, a continuous homomorphism from A onto B. We show that this is indeed the case for amenable group algebras.

Functions of free noncommuting variables and their differential calculus

Victor Vinnikov, Ben Gurion University, Israel

(Joint work with D. Kalyuzhnyi-Verbovetskii)

We define a function of d noncommuting variables as a function from a domain in $(\mathbb{C}^{n\times n})^d$ to $\mathbb{C}^{n\times n}$, for all matrix dimensions n, satisfying natural compatibility conditions for different values of n (it has to respect direct sums, and to commute with joint similarity). Our main motivation came from the work of Helton and his coworkers on matrix convexity and matrix positivity, but as it turned out variants of this notion were already considered by J. L. Taylor in his work on functional calculus for noncommuting operators back in early 1970s. The main examples are provided by polynomials and power series in noncommuting variables, and our main result is a kind of noncommutative Taylor series under very weak regularity assumptions (in fact, local boundedness more or less suffices) which shows that in many natural situations this is everything. E.g., given a noncommutative function whose entries are polynomials in matrix entries of the arguments of uniformly bounded degree (with respect to the matrix dimension), this function equals a noncommutative polynomial. To prove these things we construct a kind of noncommutative differential calculus (more precisely, this calculus combines differential calculus and the calculus of finite differences). This should have many applications — e.g. we can establish easily various foundational results on singularities of noncommutative rational functions in terms of their minimal realizations which were previously known only in special situations and required a great ammount of ingenuity and labour to establish.

Operators that are not orbit-reflexive

Jan Vršovský, Czech Academy of Sciences, Czech Republic

(Joint work with Vladimír Müller)

Let T be a bounded linear operator on a (real, complex) Banach space X. Analogously to the definition of reflexivity, we say that T is orbit-reflexive if every bounded linear operator A belongs to the closure of $\{T^n : n = 1, 2, 3\dots\}$ in the strong operator topology whenever $Au \in \overline{\{T^n u : n = 1, 2, 3\dots\}}$ for each $u \in X$. While the notion of reflexivity is connected to the problem of invariant subspaces, orbit-reflexivity is in the same way connected to the problem of invariant subsets.

Recently, Sophie Grivaux and Maria Roginskaya found a Hilbert space operator which is not orbit-reflexive. We present a different, more simple type of construction that also provides a Hilbert space operator which is not orbit-reflexive, and moreover a Banach space operator which is reflexive but not orbit-reflexive.

Discrete-time stability of polynomial matrices

Harald Wimmer, University of Würzburg, Germany

Polynomial matrices $G(z) = Iz^m - \sum_{i=0}^{m-1} C_i z^i$ with normal or hermitian coefficients C_i are studied.
Results on block diagonal stability and discrete–time Lyapunov equations are used to extend the following theorem from polynomials to polynomial matrices.
Let $g(z) = z^m - \sum_{i=0}^{m-1} c_i z^i$ be a real polynomial. Suppose $c_0 \neq 0$ and

$$\sum_{i=0}^{m-1} |c_i| \leq 1.$$

Then $\rho(g) = \max\{|\lambda|;\, g(\lambda) = 0\} \leq 1$. If λ is a root of $g(z)$ with $|\lambda| = 1$ then λ is a simple root and $\lambda^d = \pm 1$ for some d with $d \,|\, m$. If $\rho(g) = 1$ then either $g(1) = 1$ and

$$g(z) = (z^k - 1) f(z^k)$$

or $g(1) \neq 1$ and

$$g(z) = (z^k + 1) f(z^k),$$

and $f(\mu) \neq 0$ if $|\mu| = 1$.
Two applications are discussed, namely an Eneström–Kakeya theorem for polynomial matrices and a stability and convergence result for a system of difference equations.

Thoughts on the 90th birthday of Professor Ivan Vidav

Jaroslav Zemánek, Polish Academy of Sciences, Poland

We intend to look at some highlights of the work of Professor Ivan Vidav, their historical circumstances and further developments.

Themes for Working Groups

Linear algebra, commuting varieties and Hilbert schemes

R. Basili, A. Conca, T. Košir

We will discuss problems that arise when studying pairs or triples of commuting matrices. For instance, a number of authors recently discussed the question of description of the dense orbit in the nilpotent commutator of a given nilpotent matrix (Panyushev, Basili and Iarrobino, Košir and Oblak), and also more general question of describing all possible pairs of Jordan partitions for a pair of commuting nilpotent matrix. Since the participants of the workshop are coming from various backgrounds, we expect to discuss these and other problems of commuting matrices from many aspects, e.g. of linear algebra, commutative algebra, algebraic geometry, Lie theory, and combinatorics.

Higher-rank numerical ranges and applications

J. Holbrook, C.-K. Li

The study of higher rank numerical ranges was motivated by quantum error correction. It turns out the higher rank numerical range is also useful in the study of many other topics such as isotropic subspaces, matrix equations, dilation theory, perturbation of operators, matrix inequalities, preserver problems, joint numerical ranges and joint essential numerical ranges. In this working group, some recent results on the joint numerical range and its connections to other subjects will be discussed.

Matrix and operator inequalities

F. Kittaneh, C.-K. Li

Recent matrix and operator inequalities will be discussed. These include numerical radius inequalities and norm inequalities for commutators of Hilbert space operators.

Dynamical networks

M. Kramar Fijavž, R. Nagel

The term "dynamical networks" refers to the study of various dynamical processes going on on a static graph. These processes are described by a system of PDEs with boundary conditions in the vertices of the graph. In recent years we have, together with various coauthors, obtained many results regarding solvability, qualitative behavior of the solutions and controllability. The proofs (and results) intertwine continuous functional analysis with discrete mathematics.

We will present some open questions which should be of interest to specialist in linear algebra, graph theory or operator theory. Here is a sample: Which $n \times n$ positive column stochastic matrices A satisfy that the set $\{v, Av, A^2v, \dots, A^{n-1}v\}$ has full range if we take $v = e_1$, e.g. the first basis vector?

The above matrix A can be viewed as a weighted adjacent matrix of a directed graph. This condition characterizes the vertices of the graph, in which our dynamical process is controllable, but it is still unclear what are its consequences for the graph.

Patterned matrices

T. Laffey

Suppose $\sigma = (\lambda_1, \ \ldots \ , \lambda_n)$ is a list of complex numbers. We say that σ is **realizable** if σ is the spectrum of an (entrywise) nonnegative matrix. In this case, we say that A **realizables** σ. Suppose that σ satisfies all known necessary conditions for realizability. Then one seeks to prove that σ is realizable by constructing a realizing matrix. Several authors have sought realizing matrices A whose entries exhibit some nice patterns, e.g. companion matrices, circulants, upper Hessenberg matrices, M-matrices. Frequently, one seeks patterned matrices A whose characteristic polynomial is easy to express in terms of the entries of A. Most known examples are of block-companion type, and we seek other useful patterns. In particular, identifying subclasses of the class of symmetric matrices which lead to realizations of real spectra has proved very elusive. We aim to identify further useful classes.

Transitive subspaces

V. Lomonosov

A set S of operators *acts transitively* on a vector space V if for all pairs of vectors $x, y \in V$ with $x \neq 0$ there is an element $A \in S$ such that $Ax = y$. For a given positive integer k the set S *acts k-transitively* on V if for any linearly independent k vectors $\{x_1, x_2, ..., x_k\}$ and any k vectors $\{y_1, y_2, ..., y_k\}$ of V there is $A \in S$ such that $Ax_i = y_i, i = 1, ..., k$.

A classical theorem due to Burnside asserts that the only subalgebra of $M_n(\mathbb{C})$ which acts transitively on $\mathbb{C}^n$ is the matrix algebra itself. The situation is different for subspaces. For every $0 \leq k < min\{m, n\}$, there are k-transitive subspaces of $M_{mn}(\mathbb{C})$ which are not $(k+1)$-transitive. We are going to discuss transitivity questions for subspaces of linear operators with different structures.

Approximate versions of triangularizing or reducing conditions

H. Radjavi

In the past few years work has been done by several authors (including Bernik, Drnovšek, Kokol-Bukovšek, Košir, Marcoux, Mastnak, Omladič, Radjavi) in demonstrating that certain approximate versions of hypotheses in previously known results can replace the exact versions. To illustrate with a question: what can be said about a group or semigroup S of matrices if, for all A and B in S, the commutator $AB - BA$ is “small” in some sense? Bernik and Radjavi have dealt with this question when smallness is measured in terms of norm or spectral radius; Mastnak and Radjavi have studied it when smallness means that the spectrum is concentrated on one line.

Local-to-global properties of semigroups of matrices

H. Radjavi

By this is meant proving something about a whole semigroup S with available knowledge of "part" of the semigroup. One example is: if every pair of members of S is triangularizable, then S is triangularizable. A different example was given recently by Rosenthal-Radjavi: if a fixed nonzero linear functional applied to the members of S yields only a finite number of values, then S itself is finite. A third result of this sort I proved is: if a rank-one linear functional yields only real values when applied to (a complex S), then S is simultaneously similar to a semigroup of real matrices (and similar results for other fields and subfields).

Canonical forms

L. Rodman

The working group will address various aspects of the local behavior of canonical forms of matrices, matrix pairs etc., under small changes in the matrix, matrix pairs, etc. Basic canonical forms will be considered, such as the Jordan canonical form (where many results are known), and other forms, with emphasis on structures that appear in control systems. Open problems will be discussed.

Using the spectral graph theory techniques to search for strongly regular and distance regular graphs

D. Stevanović

The technique of star complements was employed successfully to several problems in recent years, for example, to complete the characterization of graphs with least eigenvalue -2. In short, a star complement for an eigenvalue λ of multiplicity m in an n-vertex graph G is any $(n-m)$-vertex induced subgraph H of G which does not have λ as an eigenvalue. The Reconstruction theorem says that, in principle (and most of the time, in practice as well), a graph can be reconstructed if we know an eigenvalue and a corresponding star complement only.

Here we want to discuss to what extent star complements may be used to determine strongly regular or distance regular graphs with a given set of parameters. While there were a few applications of star complements to strongly regular graphs already, they were restricted to characterizing maximal graphs having some small graph as a star complement (e.g., the McLaughlin graph has $6K_1 \cup K_{1,16}$ as a star complement for eigenvalue 2).

So far I have shown, together with my PhD student, that they can be applied successfully to recently characterized SRGs with parameters $(81, 20, 1, 6)$ or $(105, 32, 4, 12)$, which both have 2 as the second eigenvalue. It appears that star complement technique might be applicable to other graphs with small second eigenvalue of large multiplicity, but the true potential of its applicability to SRGs or DRGs is still rather questionable. Evenmore, there are fundamental questions that need to be resolved as well: for example, it is not yet known whether every induced subgraph which does not have λ as an eigenvalue can be extended to a star complement?