On the BLUEs in two linear models via C. R. Rao's Pandora's box

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Partitioned linear models are used in the estimations of subparameters in regression models as well as in the investigations of some submodels and reduced models associated with the original model. In this study, we consider the estimation of the parameters in two partitioned linear models, denoted by $\mathcal{A} = \{\mathbf{y}, \mathbf{X}_1\boldsymbol{\beta}_1 + \mathbf{X}_2\boldsymbol{\beta}_2, \mathbf{V}_{\mathcal{A}}\}$ and $\mathcal{B} = \{\mathbf{y}, \mathbf{X}_1\boldsymbol{\beta}_1 + \mathbf{X}_2\boldsymbol{\beta}_2, \mathbf{V}_{\mathcal{B}}\}$, which we call full models. Correspondingly, we define submodels $\mathcal{A}_1 = \{\mathbf{y}, \mathbf{X}_1\boldsymbol{\beta}_1, \mathbf{V}_{\mathcal{A}}\}$ and $\mathcal{B}_1 = \{\mathbf{y}, \mathbf{X}_1\boldsymbol{\beta}_1, \mathbf{V}_{\mathcal{B}}\}$. Using the so-called Pandora's Box approach introduced by Rao [C. R. Rao, Unified theory of linear estimation, Sankhyā Ser. A 33, 371–394 (1971)], we give new necessary and sufficient conditions for the equality between the best linear unbiased estimators (BLUEs) of $\mathbf{X}_1\boldsymbol{\beta}_1$ under $\mathcal{A}_1$ and $\mathcal{B}_1$ as well as under $\mathcal{A}$ and $\mathcal{B}$. In our considerations we will utilise the Frisch–Waugh–Lovell theorem which provides a connection between the full model $\mathcal{A}$ and the reduced model $\mathcal{A}_r = \{\mathbf{M}_2\mathbf{y}, \mathbf{M}_2\mathbf{X}_1\boldsymbol{\beta}_1, \mathbf{M}_2\mathbf{V}_{\mathcal{A}}\mathbf{M}_2\}$ with $\mathbf{M}_2$ being an appropriate orthogonal projector. Moreover, we consider the equality of the BLUEs under the full models assuming that they are equal under the submodels. We note that considering the problems of linear estimation from linear statistical models by means of the Pandora's Box approach have some advantages from the computational point of view since estimation and inference from a linear model reduces to the computation of a generalized inverse of the matrix given in the Pandora's Box equation as also noted by Rao (1971).

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