Invited ILAS Lecture

Pólya permanent problem: 100 years after

Alexander Guterman

Department of Mathematics and Mechanics Moscow State University (Russia) guterman@list.ru

Two important functions in matrix theory, determinant and permanent, look very similar:

det
$$A = \sum_{\sigma \in S_n} (-1)^{\sigma} a_{1\sigma(1)} \cdots a_{n\sigma(n)}$$
 and per $A = \sum_{\sigma \in S_n} a_{1\sigma(1)} \cdots a_{n\sigma(n)}$

here $A = (a_{ij}) \in M_n(\mathbb{F})$ is an $n \times n$ matrix and S_n denotes the set of all permutations of the set $\{1, \ldots, n\}$.

While the computation of the determinant can be done in a polynomial time, it is still an open question, if there are such algorithms to compute the permanent. Due to this reason, starting from the work by Pólya, 1913, different approaches to convert the permanent into the determinant were under the intensive investigation.

The lecture will contain the exposition of this theory during the past 100 years including our recent research results.

This talk is based on a series of joint works with M. BUDREVICH (Lomonosov Moscow State University), G. DOLINAR (University of Ljubljana), B. KUZMA (University of Primorska) and M. OREL (University of Primorska).

Invited talks

A new difference-based weighted mixed Liu estimator in partially linear models

FIKRI AKDENIZ Department of Mathematics and Computer Science Çağ University (Turkey) fikriakdeniz@gmail.com

In this paper, a generalized difference-based estimator is introduced for the vector parameter β in partially linear model when the errors are correlated. A generalized difference-based Liu estimator is defined for the vector parameter β . Under the linear stochastic constraint $r = R\beta + e$, we introduce a new generalized-based weighted mixed Liu estimator. The efficiency properties of the difference-based weighted mixed regression method is analyzed.

This is a joint work with E. AKDENIZ DURAN (Istanbul Medeniyet University).

Change detection in polarimetric SAR images using complex Wishart distributed matrices

KNUT CONRADSEN Department of Applied Mathematics and Computer Science Technical University of Denmark (Denmark) knco@dtu.dk

In surveillance it is important to be able to detect natural or man-made changes e.g. based on sequences of satellite or air borne images of the same area taken at different times, The mapping capability of synthetic aperture radar (SAR) is independent of e.g. cloud cover, and thus this technology holds a strong potential for change detection studies in remote sensing. In polarimetric synthetic aperture radar we measure the amplitude and phase of backscattered signals in four combinations of the linear horizontal and vertical receive and transmit polarizations. The-se signals form a complex scattering matrix, and after suitable preprocessing the outcome at each picture element (pixel) may be represented as a 3 by 3 Hermitian matrix following a complex Wishart distribution.

One approach to solving the change detection problem based on SAR images is therefore to apply suitable statistical tests in the complex Wishart distribution. We propose a set-up for a systematic solution to the (practical) problems using the likelihood ratio test statistics. We show some examples based on a time series of images with 1024 by 1024 pixels.

This talk reports joint work with A. AASBJERG NIELSEN (Technical University of Denmark) and H. SKRIVER (Technical University of Denmark).

Analyzing Markov chains using Kronecker products

TUĞRUL DAYAR Department of Computer Engineering Bilkant University (Turkay)

Bilkent University (Turkey) tugrul@cs.bilkent.edu.tr

Kronecker products are used to define the underlying Markov chain (MC) in various modeling formalisms, including compositional Markovian models, hierarchical Markovian models, and stochastic process algebras. Although the Kronecker representation does not provide a solution to the storage problem of state probability vectors associated with the model, it enables the storage of the underlying state transition matrix compactly, thereby facilitating the analysis of multi-dimensional models that are an order of magnitude larger than those that can be analyzed with conventional sparse matrix techniques on the same platform due to memory limitations. In the Kronecker based approach, the generator matrix underlying the MC is represented using Kronecker products of smaller matrices and is never explicitly generated. The implementation of transient and steady–state solvers rests on this compact Kronecker representation, thanks to the existence of an efficient vector– Kronecker product multiplication algorithm known as the shuffle algorithm. Here, we take a vector–matrix approach and discuss recent results related to the analysis of MCs based on Kronecker products independently from modeling formalisms.

Riesz probability distribution on symmetric matrices and extensions of the Olkin-Rubin characterization

ABDELHAMID HASSAIRI

Laboratory of Probability and Statistics Sfax University (Tunisia) abdelhamid.Hassairi@fss.rnu.tn

We introduce the Riesz probability distribution on the cone of positive symmetric matrices as a generalization of the Wishart distribution. We then define some related distributions and established some properties. Versions of the Olkin-Rubin theorem without invariance of the "quotient" are given.

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Sensitivity analysis for perfect state transfer in quantum walks

STEVE KIRKLAND Department of Mathematics University of Manitoba (Canada) stephen.kirkland@umanitoba.ca

Suppose that *G* is a graph on vertices 1,..., *n* with adjacency matrix *A*, and for each $t \ge 0$, let U(t) = exp(itA), where exp denotes the matrix exponential. Fixing an index *k* between 1 and *n*, it is straightforward to determine that $\sum_{j=1}^{n} |u_{k,j}(t)|^2 = 1$; the vectors $[|u_{k,1}(t)|^2 \dots |u_{k,n}(t)|^2]$, $t \ge 0$ can be thought of as a *continuous time quantum walk* on *G*, starting from vertex *k*. That is, $|u_{k,j}(t)|^2$ represents the probability that a quantum walk on *G* starting from vertex *k* arrives at vertex *j* at time *t*.

The quantities $|u_{k,j}(t)|^2$, j = 1, ..., n are of interest in quantum physics. For a network (represented by *G*) of interacting quantum states, a state is input at vertex *k*, and after time *t* has elapsed, the state is read out at vertex *j*. The quantity $|u_{k,j}(t)|^2$, which is the known as the *fidelity*, measures the similarity between the original state input at *k*, and the state read out at *j*. In particular, if $|u_{k,j}(t_0)|^2 = 1$ for some $t_0 > 0$, then we say there is *perfect state transfer* from *k* to *j* at time t_0 . The last decade has seen a good deal of interest in perfect state transfer, in part because it serves as a model for the transfer of information in a quantum computer.

In the setting of perfect state transfer we consider, in this talk, the sensitivity of the fidelity with respect to both the readout time t_0 , and the intensity of the interactions between spins. Using techniques from matrix analysis, we derive expressions for the derivatives of the fidelity with respect to both types of quantities. The results may help to inform the design of spin networks that not only exhibit perfect state transfer but also offer some forgiveness to errors in readout time and/or spin interactions.

Life distributions in survival analysis and reliability: Structure of nonparametric, semiparametric and parametric families

INGRAM OLKIN Department of Health Research and Policy Stanford University (USA) olkin@stanford.edu

Keywords: survival analysis, reliability theory, log concave densities, hazard rates, stochastic orders, nonparametric families, semiparametric families

In this talk I will discuss some of the characteristics of life distributions that arise in survival analysis and reliability theory. Alternative definitions of a distribution are discussed and then related to a variety of stochastic orders: hazard rate order, likelihood ratio order, convex order, Lorenz order. Nonparametric families, in particular log concave densities, completely monotone distributions, increasing hazard rate families, new-better-than-used families and bathtub hazard rate families are analyzed. A taxonomy for semiparametric families is presented and the effect of introducing parameters on various stochastic orders is shown. Finally, the introduction of covariate models in these families is developed.

On 5 \times 5 golden magic matrices and 5 \times 5 *Stifelsche Quadrate*

GEORGE P. H. STYAN Department of Mathematics and Statistics McGill University (Canada) geostyan@gmail.com

We comment on some properties of 5×5 golden magic matrices and 5×5 *Stifelsche Quadrate*. Our "golden magic matrices" **M** are 5×5 fully-magic matrices where \mathbf{M}^2 is a Toeplitz-circulant with top row (p,q,r,r,q) and eigenvalues $a + \Phi b$; here $\Phi = (1 + \sqrt{5})/2$ is the Golden Ratio and a, b are rational numbers. For example, we find that the column-flipped Agrippa–Paracelsus classic fully-magic matrix **A** and the fully-magic Latin-square Hankel-circulant **H** are golden

$$\mathbf{A} = \begin{pmatrix} 3 & 20 & 7 & 24 & 11 \\ 16 & 8 & 25 & 12 & 4 \\ 9 & 21 & 13 & 5 & 17 \\ 22 & 14 & 1 & 18 & 10 \\ 15 & 2 & 19 & 6 & 23 \end{pmatrix}, \qquad \mathbf{H} = \begin{pmatrix} 0 & 3 & 1 & 4 & 2 \\ 3 & 1 & 4 & 2 & 0 \\ 1 & 4 & 2 & 0 & 3 \\ 4 & 2 & 0 & 3 & 1 \\ 2 & 0 & 3 & 1 & 4 \end{pmatrix}.$$
(1)

We define 5×5 *Stifelsche Quadrate* as 5×5 fully-magic matrices where the 3×3 inner centre submatrix is also fully-magic, for example

$$\mathbf{S} = \begin{pmatrix} 3 & 18 & 21 & 22 & 1 \\ 24 & 16 & 11 & 12 & 2 \\ 7 & 9 & 13 & 17 & 19 \\ 6 & 14 & 15 & 10 & 20 \\ 25 & 8 & 5 & 4 & 23 \end{pmatrix}.$$
 (2)

The 5 × 5 magic square defined by **S** was chosen recently for a German postage stamp in honour of the mathematician and Augustinian monk Michael Stifel (1487–1567). The matrix **S** with magic sum m = 65 is not golden but has a nilpotent property in common with the golden matrices **A** and **H**: we find that $N_S^2 = 0$ where $N_S = S + SF - 2m\bar{E}$, with **F** the flip matrix and \bar{E} the 5 × 5 matrix with every entry equal to 1/5. Moreover, $N_A^2 = N_H^2 = 0$.

Joint research with M. A. AMELA (General Pico).

One-parameter semigroups of endomorphisms of a symmetric cone

KLEMEN ŠIVIC Faculty of Mathematics and Physics University of Ljubljana (Slovenia) klemen.sivic@fmf.uni-lj.si

Let *C* be a closed cone in a Euclidean space *V*. A linear map $A: V \to V$ is called an endomorphism of the cone *C* or a positive map if $A(C) \subseteq C$. Let $\{e^{tA}; t \ge 0\}$ be a one-parameter semigroup of endomorphisms of the cone *C*. If *C* is polyhedral, then it is well-known that the generator *A* of the semigroup can be written as a sum of an endomorphism of *C* and a generator of one-parameter group of automorphisms of *C*. It is known that such a decomposition does not exist in general, but it is not known whether it exists if the cone *C* is symmetric. i.e. homogeneous and self-dual. We answer this question negatively. Explicitly, for each symmetric cone *C* of rank at least 3 we find a generator of a one-parameter semigroup of endomorphisms of *C* that cannot be written as a sum of an endomorphism of *C* and a generator of a one-parameter group of automorphisms of *C*. The work is motivated by the study of affine processes on symmetric cones.

This is a joint work with B. KUZMA (University of Primorska), M. OMLADIČ (University of Ljubljana) and J. TEICHMANN (ETH Zürich).

The role of coupling and the deviation matrix in calculating the value of capacity for queueing systems

PETER TAYLOR Department of Mathematics and Statistics

University of Melbourne (Australia) p.taylor@ms.unimelb.edu.au

In queues with finite capacity *C*, customers are lost when they arrive to find *C* customers already present. Assuming that each arriving customer brings a certain amount of revenue, we are interested in calculating the value of an extra unit of capacity by deriving the expected amount of extra revenue earned over a finite time horizon [0, T].

There are different ways of approaching this problem. One involves the derivation of Markov renewal equations by conditioning on the first instance at which the state of the queue changes. A second involves an elegant coupling argument. We shall describe both of these approaches and the role that the deviation matrix of the Markov chain plays in the analysis.

This is joint work with P. BRAUNSTEINS (University of Melbourne) and S. HAUT-PHENNE (University of Melbourne).

Expansion formulas for inertias of quadratic matrix-valued functions with applications

YONGGE TIAN CEMA University of Finance and Economics (China) yongge.tian@gmail.com

I shall introduce how to establish expansion formulas for calculating inertias of quadratic matrix-valued functions, and present their applications in characterizing mathematical and statistical properties of estimations in regression analysis.

Semi Markov migration process in a stochastic environment in credit risk

PANAGIOTIS-CHRISTOS VASSILIOU

Mathematics department Aristole University of Thessaloniki (Greece) vasiliou@math.auth.gr

In the present the idea of stochastic Market environment comes into play to express the changes in general economy, which affects any industry in small or great amounts of turbulence. We model the evolution of the Market among its possible -states as an F-inhomogeneous semi-Markov process. This idea leads us to modeling the migration process of defaultable bonds as different *F*-inhomogeneous semi-Markov process. The survival probabilities of a defaultable bond in every credit grade are found. The asymptotic behaviour of the survival probabilities is established under certain conditions. Also, it is proved under what conditions the convergence is geometrically fast. The stochastic foundation of the general stochastic discrete-time Market is provided, by proving that the market is viable, if and only if, there exists an equivalent martingale measure, from which we construct the forward probability measure and under which the discounted default free bond price process for all possible states of the Market is a martingale. The term structure of credit spread and the change of real-world probability measure to forward probability measure are studied. In the form of a Theorem it is proved that under certain conditions, changing the real probability measure to a forward probability measure, does not affect the inhomogeneous semi Markov process modeling the migration of defaultable bonds. That is, it is proved that it only changes the basic parameters and we provide a relation among the transition probabilities under the two measures. Finally, parameter estimation and calibration of the inhomogeneous semi-Markov chain in stochastic environment is being provided.

Contributed talks

Simulating data to demonstrate that the Integrated Likelihood Method (ILM) works for parameter estimation when some data values are missing at random

PHILIP V. BERTRAND Solihull (United Kingdom) pbbg12972@blueyonder.co.uk

To demonstrate the efficacy of the method data are simulated from a multivariate normal distribution with known parameters. Then a proportion of the values are deleted at random. The resulting data are analyzed using ILM. This demonstrates that the parameter estimate obtained are consistent with the true values. This can be verified by increasing the size of the data sets and by using different parameter values.

On the BLUEs in two linear models via C. R. Rao's Pandora's box

NESRIN GÜLER Department of Statistics University of Sakarya (Turkey) nesring@sakarya.edu.tr

Partitioned linear models are used in the estimations of subparameters in regression models as well as in the investigations of some submodels and reduced models associated with the original model. In this study, we consider the estimation of the parameters in two partitioned linear models, denoted by $\mathcal{A} = \{\mathbf{y}, \mathbf{X}_1 \boldsymbol{\beta}_1 + \mathbf{X}_2 \boldsymbol{\beta}_2, \mathbf{V}_{\mathcal{A}}\}$ and $\mathcal{B} = \{\mathbf{y}, \mathbf{X}_1 \boldsymbol{\beta}_1 + \mathbf{X}_2 \boldsymbol{\beta}_2, \mathbf{V}_{\mathcal{B}}\}$, which we call full models. Correspondingly, we define submodels $\mathcal{A}_1 = \{\mathbf{y}, \mathbf{X}_1 \boldsymbol{\beta}_1, \mathbf{V}_A\}$ and $\mathcal{B}_1 = \{\mathbf{y}, \mathbf{X}_1 \boldsymbol{\beta}_1, \mathbf{V}_B\}$. Using the so-called Pandora's Box approach introduced by Rao [C. R. Rao, Unified theory of linear estimation, Sankhyā Ser. A 33, 371–394 (1971)], we give new necessary and sufficient conditions for the equality between the best linear unbiased estimators (BLUEs) of $X_1\beta_1$ under A_1 and B_1 as well as under A and B. In our considerations we will utilise the Frisch-Waugh-Lovell theorem which provides a connection between the full model \mathcal{A} and the reduced model $\mathcal{A}_r = \{\mathbf{M}_2 \mathbf{y}, \mathbf{M}_2 \mathbf{X}_1 \boldsymbol{\beta}_1, \mathbf{M}_2 \mathbf{V}_{\mathcal{A}} \mathbf{M}_2\}$ with \mathbf{M}_2 being an appropriate orthogonal projector. Moreover, we consider the equality of the BLUEs under the full models assuming that they are equal under the submodels. We note that considering the problems of linear estimation from linear statistical models by means of the Pandora's Box approach have some advantages from the computational point of view since estimation and inference from a linear model reduces to

the computation of a generalized inverse of the matrix given in the Pandora's Box equation as also noted by Rao (1971).

This is a joint work with S. PUNTANEN (University of Tampere) and H. ÖZDEMIR (University of Sakarya).

The computation of the group inverse and related properties of Markov chains via perturbations

JEFFREY J. HUNTER

School of Computer and Mathematical Sciences Auckland University of Technology (New Zealand) jeffrey.hunter@aut.ac.nz

The derivation of the group inverse and the mean first passage times in a finite ergodic Markov chain is explored. The basic technique involves row by row perturbations of the transition matrix with a systematic update at the each stage. By starting from a simple base where no formal computations are required, six different algorithms are compared for accuracy. The techniques are based on those outlined in Hunter, J. J., The computation of stationary distributions of Markov chains through perturbations, Journal of Applied Mathematics and Stochastic Analysis, 4, 29-46, (1991).

An application of the generalised JLS model on different stock market indices and the 2007–2008 financial crisis

ŠPELA JEZERNIK ŠIRCA Xlab Ljubljana (Slovenia) spela.jezernik@gmail.com

Sornette and co-workers proposed that, prior to crashes, the stock index price time series is characterised by the Log-Periodic Power Law (LPPL) equation of the Johansen–Ledoit–Sornette (JLS) model. In this paper, we use a Differential Evolution algorithm for global optimisation of the highly nonlinear JLS model. We analyse the JLS model's residuals and propose an ARMA/GARCH error model to capture the residuals' behaviour. Furthermore, we use the extended autocorrelation function (EACF) method for an order determination of the ARMA/GARCH model and compare these results with those of the Akaike and Bayesian Information Criteria. The original JLS model and its generalisation are applied to the well-documented crash of October 1987 of the indices S&P 500 and Dow Jones Industrial Average, and to the DAX index prior to the crash of 1998. We also provide empirical results to show that

these models could have been used to predict the 2007–2008 financial crisis. Moreover, we show that our generalised JLS model improves the statistical properties of the model residuals.

This is a joint work with M. OMLADIČ (University of Ljubljana).

Using the Gram-Schmidt construction to develop linear models

LYNN R. LAMOTTE Biostatistics Program LSUHSC School of Public Health (USA) llamot@lsuhsc.edu

This talk describes how the Gram-Schmidt construction can be used to simplify and unify the development of the basic linear algebra results required for statistical inference in the Gauss-Markov model.

Studying the singularity of LCM-type matrices via semilattice structures and their Möbius functions

MIKA MATTILA School of Information Sciences University of Tampere (Finland) mika.mattila@uta.fi

The invertibility of LCM matrices and their Hadamard powers have been studied a lot over the years by many authors. Bourque and Ligh conjectured in 1992 that the LCM matrix $[S] = [[x_i, x_j]]$ on any GCD closed set $S = \{x_1, x_2, \ldots, x_n\}$ is invertible, but in 1997 this was proven false by Haukkanen et al. However, currently there are many open conjectures concerning LCM matrices and their real Hadamard powers presented by Hong. In this presentation we utilize lattice-theoretic structures and the Möbius function to explain the singularity of classical LCM matrices and their Hadamard powers. At the same time we end up disproving some of Hong's conjectures. We apply the mathematics software Sage to show that every 8-element GCD closed set S, for which the LCM matrix [S] is singular, has the same semilattice structure. We also construct a GCD closed set S of odd numbers such that the LCM matrix [S] is singular. Elementary mathematical analysis is applied to prove that for most semilattice structures there exist a set $S = \{x_1, x_2, \ldots, x_n\}$ of positive integers and a real number $\alpha > 0$ such that *S* possesses this structure and the power LCM matrix $[[x_i, x_i]^{\alpha}]$ is singular.

This is a joint work with P. HAUKKANEN (University of Tampere) and J. MÄNTYSALO (University of Tampere).

NINA OTOPAL Institute of Mathematics, Physics and Mechanics Ljubljana (Slovenia) nina.otopal@gmail.com

Kernel canonical correlation analysis (KCCA) is a procedure for assessing the relationship between two sets of random variables when the classical method, canonical correlation analysis (CCA), fails because of the nonlinearity of the data. The KCCA method is mostly used in machine learning, especially for information retrieval and text mining. Because the data is often represented with non-negative numbers, we propose to incorporate the non-negativity restriction directly into the KCCA method. Similar restrictions have been studied in relation to the classical CCA and called restricted canonical correlation analysis (RCCA), so that we call the proposed method *restricted kernel canonical correlation analysis* (RKCCA).

With the Karush-Kuhn-Tucker theorem we show that the solution of RKCCA equals an unconstrained solution to a modified CCA problem on two random vectors with known covariance matrix where one or several variables have been excluded. Furthermore we use the idea of sub-vectors and sub-matrices to translate the problem of searching for the kernel canonical correlation under non-negativity restriction into an optimization problem related to eigenvalues of some generalized eigenvalue problem with a real symmetric matrix and a positive definite matrix.

Random walks relative to multiple transition matrices

ANTONIJA PRŠLJA Arctur d.o.o. Nova Gorica (Slovenia) antonija.prslja@arctur.si

In discrete time 0, 1, ..., k a particle is travelling between a finite set of states relative to a transition matrix and constitutes a random walk of length k. Given the cost matrix corresponding to transitions between states, the mean of the cost along a random walk of length k starting at some specified state needs to be computed in many applications. In this talk, we first introduce a generalization of the above model for multiple transition and cost matrices, and then propose Monte Carlo techniques to get approximation of the mean by using random numbers and simulation. Experiments on artificial data are conducted to evaluate the performance of the presented approaches in comparison with the one that uses diffusion wavelets method for computing powers of matrices.

SIMO PUNTANEN School of Information Sciences University of Tampere (Finland) simo.puntanen@uta.fi

The linear mixed model has strong links with a particular augmented linear model including only fixed effects. This goes back to Henderson's mixed model equations. In this talk we point out that the connection between the two models is actually very straightforward: a mixed linear model can be obtained from the augmented model by a simple linear transformation. This immediately opens up a new viewpoint for studying the relationship between the BLUEs and BLUPs in the two models.

This is a joint work with B. ARENDACKÁ (Physikalisch-Technische Bundesanstalt).

The minimax copulas

NINA RUŽIĆ Faculty of Mathematics and Physics University of Ljubljana (Slovenia) nina.ruzic@fmf.uni-lj.si

The theory of copulas started with Sklar's theorem proposing a universal model for expressing dependence of random variables. With the range of applications in applied mathematics expanding and varying from mathematics of finance to system theory, there is a growing need for new types of copulas that could serve as appropriate models in these applications. It is our aim to set a counterpart to the famous Marshall copulas (an extension of Marshall-Olkin copulas) that are typically applied to model lifetime of a two-component system where components are subject to "shocks". Even a small but essential change in the problem that the model is applied to such a system with one of the components having a backup option leads to possibly quite different copulas. So, it is our goal to construct copulas that model dependence of random variables $U = \max{X, Z}$ and $V = \min{Y, Z}$ where X, Y and Z are independent random variables. We will present a full study of the augmented case by introducing a new family of copulas, called minimax copulas, together with some of their properties and examples.

This is a joint work with M. OMLADIČ (University of Ljubljana).

On a new family of weighted total least-squares algorithms for EIV-models with arbitrary dispersion matrices

BURKHARD SCHAFFRIN

Div. of Geodetic Science. School of Earth Sciences The Ohio State University (USA) schaffrin.1@osu.edu

For a long time, algorithms for the Total Least-Squares (TLS) solution within Errors-In-Variables (EIV) Models would only tolerate "element-wise weighting," which essentially amounts to the use of diagonal dispersion matrices without auto- or cross-correlations. This dilemma was overcome first by Schaffrin and Wieser (2008), and later by Fang (2011) as well as Mahboub (2012), who all allowed to handle more general dispersion matrices, while assuming invertibility and/or lack of crosscovariances.

Finally, in his PhD dissertation, K. Snow (2012) designed an algorithm that would generate the TLS solution even if the dispersion matrices are singular and cross-covariances exist, as long as a certain uniqueness criterion is fulfilled. Here, a new but related family of algorithms will be presented that are able to generate the (properly weighted) TLS solution with greater efficiency.

This is a joint work with K. SNOW (The Ohio State University and Topcon Positioning Systems).

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Inheritance properties of generalized Schur complements and principal pivot transforms of matrices

K.C. SIVAKUMAR

Department of Mathematics Indian Institute of Technology Madras Chennai (India) kcskumar@iitm.ac.in

The notions of Schur complement and principal pivot transforms have been studied quite extensively in matrix theory. The importance of the Schur complement in problems arising in areas including numerical analysis and statistics is well documented in the literature. The notion of the principal pivot transform appears to have its roots in the theory of the linear complementarity problem. Many results on the preservation of matrix classes like the *P*-property are well known. Studies also have been extended to the case of the Moore-Penrose inverse in the definitions of the Schur complement and the principal pivot transform. In this talk, we report new results on the inheritance properties of these generalizations, in the context of certain matrix classes.

This is a joint work with K. BISHT (IIT Madras) and G. RAVINDRAN (ISI Chennai).

Enhancing Gibbs sampling method for motif finding in DNA with initial graph representation of sequences

ŽIVA STEPANČIČ

Arctur d.o.o. Nova Gorica (Slovenia) ziva.stepancic@gmail.com

Finding short patterns with residue variation in a set of sequences is still an open problem in genetics, since motif finding techniques on DNA and protein sequences are inconclusive on real data sets and their performance varies on different species. Hence finding new algorithms and evolving established methods are vital to further understand genome properties and the mechanisms of protein development. In this talk we present an approach to search for possible motifs in connection to Gibbs sampling method. Starting points in the search space are partly determined via graphical representation of input sequences opposed to completely random initial points with the standard Gibbs sampling. Our algorithm is evaluated on synthetic as well as on real data sets by using several statistics, such as sensitivity, positive predictive value, specificity, performance and correlation coefficient. Additionally, a comparison between our algorithm and basic standard Gibbs sampling algorithm is made to show improvement in accuracy, repeatability and performance.

Estimation of the covariance matrix based on two types of the forward search algorithm

ALEŠ TOMAN Faculty of Economics University of Ljubljana (Slovenia) ales.toman@ef.uni-lj.si

Multivariate normal distribution is a crucial assumption in many statistical models and estimation of the population covariance matrix is usually the first key step in modelling multivariate data. The standard sample covariance matrix is commonly used but it gives unreliable estimates if the normal distribution does not fit the data.

Forward search algorithm is an iterative and graphical method for data exploration and robust parameter estimation [1]. The algorithm orders the data according to their distances form the underlying distribution or model. The nearest observations form the initial basic set, which is very robust. The basic set is then increased in size step by step until all observations are included. Parameter estimates and different statistics are computed with the basic sets of increasing sizes.

We will demonstrate the use of the forward search algorithm in the context of robust covariance matrix and confirmatory factor model estimation [2,3]. The former uses Cook's distance to measure the influence of an observation and the later uses observational residuals. The comparison of the two methods applied to real and simulated data sets will show that outliers and influential observations can be model specific. Model-based robust techniques should therefore be emphasized.

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Solving a 6 x 6 Survo puzzle using matrix combinatorial products

KIMMO VEHKALAHTI Department of Social Research

University of Helsinki (Finland) kimmo.vehkalahti@helsinki.fi

We apply a new computational method for solving a demanding 6×6 Survo puzzle with binary matrices that are recoded and combined using the Hadamard, Kronecker, and Khatri–Rao products. An extra challenge is provided by readily given numbers that make the puzzle solvable.

This is a joint work with R. SUND (University of Helsinki).

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On *R*² **in linear mixed models**

Julia Volaufova

Biostatistics Program LSUHSC School of Public Health (USA) jvolau@lsuhsc.edu

The statistic R^2 in fixed-effects regression settings is mostly understood as a measure of the proportion of variability explained by the model. Here we look into several definitions of generalizations of R^2 defined for linear mixed effects models published during recent decades. We try to address questions such as "Do these measures coincide in specific models?" "What do they measure?", etc.

This is joint work with O. BLAHA (LSUHSC School of Public Health) and L. R. LAM-OTTE (LSUHSC School of Public Health).

Some results on permutations of matrix products

HANS JOACHIM WERNER

Institute for Financial Economics and Statistics University of Bonn (Germany) hjw.de@uni-bonn.de

It is well-known that trace(AB) ≥ 0 for real-symmetric nonnegative definite matrices A and B. However, trace(ABC) can be positive, zero or negative, even when C is real-symmetric nonnegative definite. The genesis of the present investigation is consideration of a product $A = A_1A_2 \cdots A_n$ of square matrices. Permuting the factors of A leads to a different matrix product. We are interested in conditions under which the spectrum remains invariant. The main results are for square matrices over an arbitrary algebraically closed commutative field. The special case of real-symmetric, possibly nonnegative definite, matrices is also considered.

This is a joint work with I. OLKIN (Stanford University).